UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER

PROGRAMME: B.SC. IN AGRICULTURE IV (AEM OPTION)

COURSE CODE: AEM 401

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

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QUESTION 1

(a) Draw a *schematic diagram* to show the procedure to be followed when *testing* an *economic theory*. [10 marks]

(c) List and discuss three (3) *large sample* properties of estimators. [10 marks]

(c) List and discuss the underlying assumptions of the multiple regression model. [20 marks]

QUESTION 2

A random sample of ten (10) families had the following income and food expenditure (in £ per week):

Families	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Family Income (X)	20	30	33	40	15	13	26	38	35	43
Family Expenditure (Y)	7	9	8	11	5	4	8	10	9	10

Source: Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 67

Intermediate results

$\sum X = 293$ $\sum Y = 81$ $\sum X^2 = 9{,}577$ $\sum Y^2 = 701$

$\sum XY = 2{,}574$

(a) Estimate the regression line for *food expenditure* on *income*. [10 marks]

(b) Calculate the coefficient of determination, R^2. Conduct a test of significance of this coefficient at the 5% level of significance. Provide an economic interpretation for the results of your test. [**N.B.**: You need not construct the relevant table for the test.] [10 marks]

(c) Calculate the standard errors of the estimated parameters and conduct tests of significance using the *standard error test*. If appropriate, give an *economic interpretation* for the *regression coefficient*. [10 marks]

QUESTION 3

The following table shows the values of expenditure on clothing (Y), total expenditure (X_1) and the price of clothing (X_2).

Year	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
Y	3.5	4.3	5	6	7	9	8	10	12	14
X_1	15	20	30	42	50	54	65	72	85	90
X_2	16	13	10	7	7	5	4	3	3.5	2

Source: Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 138

Intermediate results

$\sum Y = 78.8$ $\sum X_1 = 523$ $\sum X_2 = 70.5$

$\sum X_1 Y = 4{,}896.5$ $\sum X_1^2 = 33{,}439$ $\sum X_1 X_2 = 2{,}667.5$

$\sum X_2 Y = 429.9$ $\sum Y^2 = 725.74$ $\sum X_2^2 = 289.25$

(a) Find the least squares regression equation of Y on X_1 and X_2. [10 marks]

(b) Compute the coefficient of *multiple determination* and the *standard errors* of the *regression coefficients* and conduct tests of significance. [10 marks]

(c) Construct 95% confidence intervals for the population regression coefficients and provide their economic meaning in this problem. [10 marks]

QUESTION 4

The following results were obtained from a sample of 15 firms of the chemical industry on their output (Y), labour input (X_1) and capital input (X_2), measured in arbitrary units.

$\sum Y^* = 86.159$ $\sum Y^{*2} = 500.181$ $\sum Y^* X_1^* = 638.339$ $\sum X_1^* = 110.885$
$\sum X_1^{*2} = 820.118$ $\sum Y^* X_2^* = 553.851$ $\sum X_2^* = 95.966$ $\sum X_2^{*2} = 615.701$
$\sum X_1^* X_2^* = 710.138$

Here,

$Y^* = \log_e Y$
$X_1^* = \log_e X_1$
$X_2^* = \log_e X_2$

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P.139

QUESTION 4 (Continued)

(a) Fit a Cobb-Douglas production function

$$Y = \beta_o X_1^{\beta 1} X_2^{\beta 2} e^u,$$

using the above results [10 marks]

(b) Conduct tests of significance of the intercept and theconstat elasticities and provide their economic meaning. [20 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X \sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1 \overline{X}$$

$$r^2 = \hat{\beta}_1^2 \frac{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y \sum Y\right)}, \qquad F = \frac{r^2}{1 - r^2}(n-2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u^2 \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\sigma_u^2 \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \dfrac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \dfrac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$t = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \dfrac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \dfrac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

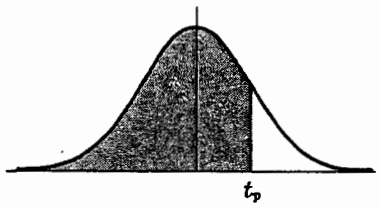
$$\text{Total SS} = \sum Y^2 - n\overline{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\overline{Y}^2,$$

$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n - k - 1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n - k - 1} = \frac{\text{Total SS - Regression SS}}{n - k - 1},$$

$$\hat{\sigma}_{\left(\hat{\beta}_j\right)} = \sqrt{(\text{j}+1)\text{th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0,1,\ldots,k.$$

Percentile Values (t_p)
for
Student's t Distribution
with ν Degrees of Freedom
(shaded area = p)



ν	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126

Source: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (5th edition), Table III, Oliver and Boyd Ltd., Edinburgh, by permission of the authors and publishers.

Appendix E Points for the Distribution of F [5% (light type) and 1% (bold face type)]

f_2	f_1 Degrees of freedom (for greater mean square)																								f_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
1	161 **4,052**	200 **4,999**	216 **5,403**	225 **5,625**	230 **5,764**	234 **5,859**	237 **5,928**	239 **5,981**	241 **6,022**	242 **6,056**	243 **6,082**	244 **6,106**	245 **6,142**	246 **6,169**	248 **6,208**	249 **6,234**	250 **6,261**	251 **6,286**	252 **6,302**	253 **6,323**	253 **6,334**	254 **6,352**	254 **6,361**	254 **6,366**	1
2	18.51 **98.49**	19.00 **99.00**	19.16 **99.17**	19.25 **99.25**	19.30 **99.30**	19.33 **99.33**	19.36 **99.36**	19.37 **99.37**	19.38 **99.39**	19.39 **99.40**	19.40 **99.41**	19.41 **99.42**	19.42 **99.43**	19.43 **99.44**	19.44 **99.45**	19.45 **99.46**	19.46 **99.47**	19.47 **99.48**	19.47 **99.48**	19.48 **99.49**	19.49 **99.49**	19.49 **99.49**	19.50 **99.50**	19.50 **99.50**	2
3	10.13 **34.12**	9.55 **30.82**	9.28 **29.46**	9.12 **28.71**	9.01 **28.24**	8.94 **27.91**	8.88 **27.67**	8.84 **27.49**	8.81 **27.34**	8.78 **27.23**	8.76 **27.13**	8.74 **27.05**	8.71 **26.92**	8.69 **26.83**	8.66 **26.69**	8.64 **26.60**	8.62 **26.50**	8.60 **26.41**	8.58 **26.35**	8.57 **26.27**	8.56 **26.23**	8.54 **26.18**	8.54 **26.14**	8.53 **26.12**	3
4	7.71 **21.20**	6.94 **18.00**	6.59 **16.69**	6.39 **15.98**	6.26 **15.52**	6.16 **15.21**	6.09 **14.98**	6.04 **14.80**	6.00 **14.66**	5.96 **14.54**	5.93 **14.45**	5.91 **14.37**	5.87 **14.24**	5.84 **14.15**	5.80 **14.02**	5.77 **13.93**	5.74 **13.83**	5.71 **13.74**	5.70 **13.69**	5.68 **13.61**	5.66 **13.57**	5.65 **13.52**	5.64 **13.48**	5.63 **13.46**	4
5	6.61 **16.26**	5.79 **13.27**	5.41 **12.06**	5.19 **11.39**	5.05 **10.97**	4.95 **10.67**	4.88 **10.45**	4.82 **10.29**	4.78 **10.15**	4.74 **10.05**	4.70 **9.96**	4.68 **9.89**	4.64 **9.77**	4.60 **9.68**	4.56 **9.55**	4.53 **9.47**	4.50 **9.38**	4.46 **9.29**	4.44 **9.24**	4.42 **9.17**	4.40 **9.13**	4.38 **9.07**	4.37 **9.04**	4.36 **9.02**	5
6	5.99 **13.74**	5.14 **10.92**	4.76 **9.78**	4.53 **9.15**	4.39 **8.75**	4.28 **8.47**	4.21 **8.26**	4.15 **8.10**	4.10 **7.98**	4.06 **7.87**	4.03 **7.79**	4.00 **7.72**	3.96 **7.60**	3.92 **7.52**	3.87 **7.39**	3.84 **7.31**	3.81 **7.23**	3.77 **7.14**	3.75 **7.09**	3.72 **7.02**	3.71 **6.99**	3.69 **6.94**	3.68 **6.90**	3.67 **6.88**	6
7	5.59 **12.25**	4.74 **9.55**	4.35 **8.45**	4.12 **7.85**	3.97 **7.46**	3.87 **7.19**	3.79 **7.00**	3.73 **6.84**	3.68 **6.71**	3.63 **6.62**	3.60 **6.54**	3.57 **6.47**	3.52 **6.35**	3.49 **6.27**	3.44 **6.15**	3.41 **6.07**	3.38 **5.98**	3.34 **5.90**	3.32 **5.85**	3.29 **5.78**	3.28 **5.75**	3.25 **5.70**	3.24 **5.67**	3.23 **5.65**	7
8	5.32 **11.26**	4.46 **8.65**	4.07 **7.59**	3.84 **7.01**	3.69 **6.63**	3.58 **6.37**	3.50 **6.19**	3.44 **6.03**	3.39 **5.91**	3.34 **5.82**	3.31 **5.74**	3.28 **5.67**	3.23 **5.56**	3.20 **5.48**	3.15 **5.36**	3.12 **5.28**	3.08 **5.20**	3.05 **5.11**	3.03 **5.06**	3.00 **5.00**	2.98 **4.96**	2.96 **4.91**	2.94 **4.88**	2.93 **4.86**	8
9	5.12 **10.56**	4.26 **8.02**	3.86 **6.99**	3.63 **6.42**	3.48 **6.06**	3.37 **5.80**	3.29 **5.62**	3.23 **5.47**	3.18 **5.35**	3.13 **5.26**	3.10 **5.18**	3.07 **5.11**	3.02 **5.00**	2.98 **4.92**	2.93 **4.80**	2.90 **4.73**	2.86 **4.64**	2.82 **4.56**	2.80 **4.51**	2.77 **4.45**	2.76 **4.41**	2.73 **4.36**	2.72 **4.33**	2.71 **4.31**	9
10	4.96 **10.04**	4.10 **7.56**	3.71 **6.55**	3.48 **5.99**	3.33 **5.64**	3.22 **5.39**	3.14 **5.21**	3.07 **5.06**	3.02 **4.95**	2.97 **4.85**	2.94 **4.78**	2.91 **4.71**	2.86 **4.60**	2.82 **4.52**	2.77 **4.41**	2.74 **4.33**	2.70 **4.25**	2.67 **4.17**	2.64 **4.12**	2.61 **4.05**	2.59 **4.01**	2.56 **3.96**	2.55 **3.93**	2.54 **3.91**	10
11	4.84 **9.65**	3.98 **7.20**	3.59 **6.22**	3.36 **5.67**	3.20 **5.32**	3.09 **5.07**	3.01 **4.88**	2.95 **4.74**	2.90 **4.63**	2.86 **4.54**	2.82 **4.46**	2.79 **4.40**	2.74 **4.29**	2.70 **4.21**	2.65 **4.10**	2.61 **4.02**	2.57 **3.94**	2.53 **3.86**	2.50 **3.80**	2.47 **3.74**	2.45 **3.70**	2.42 **3.66**	2.41 **3.62**	2.40 **3.60**	11
12	4.75 **9.33**	3.88 **6.93**	3.49 **5.95**	3.26 **5.41**	3.11 **5.06**	3.00 **4.82**	2.92 **4.65**	2.85 **4.50**	2.80 **4.39**	2.76 **4.30**	2.72 **4.22**	2.69 **4.16**	2.64 **4.05**	2.60 **3.98**	2.54 **3.86**	2.50 **3.78**	2.46 **3.70**	2.42 **3.61**	2.40 **3.56**	2.36 **3.49**	2.35 **3.46**	2.32 **3.41**	2.31 **3.38**	2.30 **3.36**	12
13	4.67 **9.07**	3.80 **6.70**	3.41 **5.74**	3.18 **5.20**	3.02 **4.86**	2.92 **4.62**	2.84 **4.44**	2.77 **4.30**	2.72 **4.19**	2.67 **4.10**	2.63 **4.02**	2.60 **3.96**	2.55 **3.85**	2.51 **3.78**	2.46 **3.67**	2.42 **3.59**	2.38 **3.51**	2.34 **3.42**	2.32 **3.37**	2.28 **3.30**	2.26 **3.27**	2.24 **3.21**	2.22 **3.18**	2.21 **3.16**	13