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UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER

PROGRAMME: B.SC. IN AGRICULTURE IV (AEM OPTION)

COURSE CODE: AEM 401

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

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QUESTION 1

(a) State with reason whether the following statements are true, false, or uncertain. Be precise.
(2 marks each) [20 marks total]

(i) The t test of significance for the regression coefficients of a three-variable regression model requires that the sampling distributions of estimators of β_0 and β_1 follow the normal distribution.

(ii) Even though the disturbance term in the Classical Linear Regression Model (CLRM) is not normally distributed, the OLS estimators are still unbiased.

(iii) If there is no intercept in the regression model, the estimated u_i will not sum to zero.

(iv) The p value and the size of a test statistic mean the same thing.

(v) In a regression model that contains the intercept, the sum of the residuals is always zero.

(vi) If a null hypothesis is not rejected, it is true.

(vii) The higher is the value of σ^2, the larger is the variance of the estimator of β_1.

(viii) The conditional and unconditional means of a random variable are the same things.

(ix) In the two-variable Population Regression Function (PRF), if the slope coefficient β_1 is zero, the intercept β_0 is estimated by the sample mean (mean of Y).

(x) The conditional variance $\text{var}(Y_i|X_i) = \sigma^2$, and the unconditional variance of Y, $\text{var}(Y) = \sigma_Y^2$ will be the same if X had no influence on Y.

(b) Discuss the possible criteria for fitting a line, stating the advantages and/or disadvantages of each. [10 marks]

(c) The deviations of the observations from the regression line may be attributed to several factors. *List* and *briefly discuss five* (5) of these factors. [10 marks]

QUESTION 2

The following table includes the gross national product (X) and the demand for food (Y) measured in arbitrary units, in an underdeveloped country over the 10-year period 1960-1969.

Year	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**	**1970**
Y	6	7	8	10	8	9	10	9	11	10
X	50	52	55	59	57	58	62	65	68	70

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 98

Intermediate results

$\sum X = 596$ $\sum Y = 88$ $\sum X^2 = 35,916$ $\sum Y^2 = 796$

$\sum XY = 5,325$

(a) *Estimate* the *food function.* [13 marks]

(b) Compute the *standard error* of the *estimate* of the *regression coefficient* and conduct a test of significance at the 5 per cent level of significance. If appropriate, interpret the results. [10 marks]

(c) Find the 99 per cent confidence interval for the population (true) regression coefficient. [7 marks]

QUESTION 3

The quantity supplied of a commodity X is assumed to be a linear function of the price of x and the wage rate of labour used in the production of x. The population supply equation is given as

$$Q = \beta_o + \beta_1 P_x + \beta_2 W + U$$

where Q = quantity supplied of x
P_x = price of x
W = wage rate
U = random error term

The data in the table below were obtained from a sample of the population.

Y=Q	20	35	30	47	60	68	76	90	100	105	130	140	125	120	135
X_1=P_x	10	15	21	26	40	37	42	33	30	38	60	65	50	35	42
X_2=W	12	10	9	8	5	7	4	5	7	5	3	4	3	1	2

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 606

Intermediate results:

$\sum Y = 1,281$ $\sum X_1 = 544$ $\sum X_2 = 85$

$\sum X_1 Y = 53,665$ $\sum X_1^2 = 22,922$ $\sum X_1 X_2 = 2,568$

$\sum X_2 Y = 5,706$ $\sum Y^2 = 132,609$ $\sum X_2^2 = 617$

Using the above sample data,

(a) Estimate the parameters by the Ordinary Least Squares (OLS) method. [13 marks]

(b) What percentage of the total variation in the quantity supplied is explained by both price of x (P_x) and wage rate (W)? [3 marks]

(c) Test the statistical significance of the partial regression coefficients for price of x (P_x) and wage rate (W). [10 marks]

(d) Compute the price elasticity of supply at the mean price and mean quantity traded. [4 marks]

QUESTION 4

From the following sample of ten (10) yearly observations a researcher wants to estimate the demand function for second-hand T.V. sets.

No. of T.V. sets (Y)	**543**	**580**	**618**	**695**	**724**	**812**	**887**	**991**	**1186**	**1940**
Price (in $) ($X$)	61	54	50	43	38	36	28	23	19	10

Source: Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 602

Intermediate results

Let $\text{Log}_e X = X^*$
$\text{Log}_e Y = Y^*$.

Then,

$\sum X^* = 34.7089$ $\sum Y^* = 67.2502$ $\sum X^{*2} = 123.242$ $\sum Y^{*2} = 453.5887$

$\sum X^* Y^* = 231.5054$

(a) Estimate the *constant elasticity demand function*

$$Y = bo \cdot X^{b1} \cdot e^{u}$$ [10 marks]

(b) Compute the coefficient of determination, r^2. Test the significance of this coefficient and interpret its specific economic meaning in this problem. [10 marks]

(c) Compute the standard error of the *constant elasticity* b_1. Test the significance of b_1 and interpret its specific economic meaning in this problem. [10 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X\sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1\overline{X}$$

$$r^2 = \hat{\beta}_1^2 \frac{\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y\sum Y\right)}, \qquad F = \frac{r^2}{1-r^2}(n-2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u^2 \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

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$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^2 \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

$$\text{Total SS} = \sum Y^2 - n\bar{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\bar{Y}^2,$$

$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n-k-1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n-k-1} = \frac{\text{Total SS - Regression SS}}{n-k-1},$$

$$\hat{\sigma}_{(\hat{\beta}_j)} = \sqrt{\text{(j+1)th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0,1,\ldots,k.$$

******Insert F-table here******