94

SUPP. 2005/2006

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER

PROGRAMME: B.SC. IN AGRICULTURE IV (AEM OPTION)

COURSE CODE: AEM 401

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.

2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE CHIEF INVIGILATOR

QUESTION 1

(a) Under what circumstances is the Z-test appropriate for testing the significance of the estimates of regression coefficients in a simple linear regression problem (i.e., with only one explanatory variable) ? [10 marks]

(b) Provide an argument for choosing the Student's t-test in favour of the Z-test in the context of the simple linear regression model. [10 marks]

(c) Discuss the student's t-test in the context of the simple linear regression model. [10 marks]

(d) What is the relationship between the Student's t-test and the standard-error test ? [10 marks]

QUESTION 2

The following table includes the gross national product (X) and the demand for food (Y) measured in arbitrary units, in an underdeveloped country over the ten-year period 1960-1969.

Year	Y	X
1960	6	50
1961	7	52
1962	8	55
1963	10	59
1964	8	57
1965	9	58
1966	10	62
1967	9	65
1968	11	68
1969	10	70

(a) Estimate the food function

$$Y = \beta_0 + \beta_1 X + U.$$ [10 marks]

(b) Calculate the coefficient of determination R^2. Conduct a test of significance of this coefficient at the 5% level of significance. Provide an economic interpretation of the results of your test. [**Note:** You need not show a complete analysis of variance table for the test.]. [10 marks]

(c) Calculate the standard errors of the estimated parameters and conduct tests of significance using the standard-error test. Use the results of your test to give an economic interpretation of the regression coefficient. [10 marks]

QUESTION 3

The following table shows the values of expenditure on clothing (Y), total expenditure (X_1) and the price of clothing (X_2).

Year	Expenditure on clothing Y	Total expenditure X_1	Price of clothing X_2
1960	3.5	15	16
1961	4.3	20	13
1962	5	30	10
1963	6	42	7
1964	7	50	7
1965	9	54	5
1966	8	65	4
1967	10	72	3
1968	12	85	3.5
1969	14	90	2

(a) Fit a non-linear function of the constant elasticity type

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} e^U$$ [15 marks]

(b) Conduct a test of the overall significance of the regression at the 5% level of significance using the analysis of variance table. Provide an economic interpretation of the results of your test. [15 marks]

QUESTION 4

(a) Compare *regression analysis* with the *analysis of variance*. [20 marks]

(b) Explain, in detail, why the coefficient of multiple determination has to be adjusted, including the comparison of the adjusted with the unadjusted coefficient. [10 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X \sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1 \overline{X}$$

$$r^2 = \hat{\beta}_1^{\,2} \frac{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y \sum Y\right)}, \qquad F = \frac{r^2}{1 - r^2}(n-2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\sigma_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$t = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n \leq 30$

$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n \leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

$$\text{Total SS} = \sum Y^2 - n\overline{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\overline{Y}^2,$$

$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n - k - 1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n - k - 1} = \frac{\text{Total SS - Regression SS}}{n - k - 1},$$

$$\hat{\sigma}_{(\hat{\beta}_j)} = \sqrt{(j+1)\text{th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0, 1, \ldots, k.$$

******Insert F-table here******

******Insert t-table here******