79

1ST SEM. 2006/2007

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER

PROGRAMME: B.SC. AG. ECON. & AGBMNGT. YEAR 3 (NEW PROG.)

COURSE CODE: AEM 203

TITLE OF PAPER: INTRO. TO MATHEMATICS FOR ECONOMISTS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE THREE QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 20 MARKS EACH.

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Question 1

a. Use Cramer's rule to find the values of x, y, and z that solve the following three equations simultaneously. [10 marks]

$$\begin{aligned} 4x + 3y - 2z &= 7 \\ x + y \qquad &= 5 \\ 3x + \qquad Z &= 4 \end{aligned}$$

b. Solve the three equations in the previous problem by using matrix inversion. [10 marks]

c. Find the following definite integrals. [10 marks]

i. $\int_1^3 3x^{1/2}dx$
ii. $\int_2^3 (e^{2x} + e^x)dx$
iii. $\int_0^\infty e^{-rt}dt$ where $r > 0$ is a constant

d. For the production function $f(K, L) = 9K^{1/3}L^{2/3}$, find the marginal products of K and L (i.e. the partial derivatives of the function with respect to K and with respect to L). [10 marks]

Question 2

a. Determine the maximum final demand which can be met in the following situation shown in the table. [10 marks]

	Input to		Level of
	Industry 1	Industry 2	Output
Industry 1	200	300	1,500
Industry 2	500	100	2,500

b. What final demand can be met when the level of output of Industry 1 is increased to 2,000 units in a situation which is in all other respects identical to that given in a. above. [10 marks]

81

Question 3

Consider the system of equations

$$xu^3 + v = y^2$$
$$3uv - x = 4$$

a. Take the differentials of both equations and solve for du and dv in terms of dx and dy. [10 marks]

b. Find $\partial u/\partial x$ and $\partial v/\partial x$ using your result in part (a). [10 marks]

Question 4

a. The output of a good is $x^a y$, where x and y are the amounts of two inputs and $a > 1$ is a parameter. A government-controlled firm is directed to maximize output subject to meeting the constraint $2x + y = 12$.

i. Solve the firm's problem. [5 marks]

ii. Use the envelope theorem to find how the maximal output changes as the parameter a varies. [5 marks]

a. In the theory of auctions, an initial value problem of the form

$$x'(t)G(t) + x(t)G'(t) = tG'(t) \text{ with } x(t_0) = t_0,$$

where G is a known function, arises. (The variable t is interpreted as a player's valuation of the object for sale in the auction.) Solve this equation, expressing the solution in terms of the function G (but not its derivative). (If you start by converting the equation into the standard form and finding the integrating factor, reflect afterwards on the fact that you did not need to do so.) [10 marks]

82

Question 5

Consider the following problem:

Maximize $Z = 2X_1 + X_2$,

subject to

$$\begin{aligned} X_2 &\leq 10 \\ 2X_1 + 5X_2 &\leq 60 \\ X_1 + X_2 &\leq 18 \\ 3X_1 + X_2 &\leq 44 \end{aligned}$$

and

$X_1 \geq 0,\ X_2 \geq 0.$

i Construct the initial simplex tableau, introducing slack variables and so forth as needed for applying the simplex method. [10 marks]

ii Solve the problem by simplex method. [10 marks]

Question 6

a. For each of the following payoff tables, determine the optimal strategy for each player by successively eliminating dominated strategies. (Indicate the order in which you eliminated strategies) [10 marks]

i.

		II		
		1	2	3
I	1	-3	1	2
	2	1	2	1
	3	1	0	-3

ii.

		II		
		1	2	3
I	1	0	4	1
	2	-1	-2	3
	3	1	3	2

b. Consider the game having the following payoff table:

		II			
		1	2	3	4
	1	5	0	3	1
I	2	2	4	3	2
	3	3	2	0	4

Use the approach described in the lecture to formulate the problem of finding the optimal mixed strategies according to the minimax criterion as a *linear programming* problem.

[10 marks]