# UNIVERSITY OF SWAZILAND

## SUPPLEMENTARY EXAMINATION PAPER

**PROGRAMME:** B.SC. AG. ECON. & AGBMNGT. YEAR 3 (NEW PROG.)

**COURSE CODE:** AEM 203

**TITLE OF PAPER:** INTRO. TO MATHEMATICS FOR ECONOMISTS

**TIME ALLOWED:** TWO (2) HOURS

**INSTRUCTION:**

1. ANSWER QUESTION ONE AND CHOOSE THREE QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 20 MARKS EACH.

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## Question 1

a. Use Cramer's rule to find the values of $x$ and $y$ that solve the following two equations simultaneously. [10 marks]

$3x - 2y = 11$
$2x + y = 12$

b. Solve the two equations in the previous problem by using matrix inversion. [10 marks]

c. Find the first derivatives of [10 marks]

i. $2x^4 + 3x^{1/2} + 7$
ii. $4/x^2$
iii. $(x + 3)/(x^2 + x)$
iv. $x\ln x$

c. Find the following indefinite integrals [10 marks]

i. $\int x^4 dx$
ii. $\int 2e^{-2x} dx$
iii. $\int (x + 3)(x + 1)^{1/2} dx$
iv. $\int (4x + 2)/(x^2 + x) dx$

## Question 2

Determine the level of output which is necessary to meet final demands of 1,000 and 2,000 respectively when the technological coefficients are given by the following matrices. [20 marks]

a. $\begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.1 & 0.6 \\ 0.4 & 0.1 \end{bmatrix}$

**Question 3**

The equilibrium values of the variables $Y$ and $r$ are given by the solution of the two equations

$$I(r) = S(Y)$$
$$aY + L(r) = M$$

where $a > 0$ and $M$ are parameters, $I$ is an increasing differentiable function, and $S$ and $L$ are decreasing differentiable functions. How do $Y$ and $r$ change when $M$ increases (holding $a$ constant)? [20 marks]

**Question 4**

a. A firm's output depends upon the amount $x$ of an input and a parameter $a$, according to the function $f(x, a)$. This function $f$ is increasing in $a$. The price of the firm's output is $p > 0$ and the price of the input is $w > 0$. Determine the sign of the derivative of the firm's *maximal* profit with respect to $a$. [10 marks]

b. Solve the differential equation $x''(t) - 4x'(t) + 4x(t) = 5$ in general, and subject to the initial conditions $x(0) = 4$ and $x'(0) = 6$. [10 marks]

**Question 5**

a. Find the general solution of the system of equations [10 marks]

$$x'(t) = 4y(t)$$
$$y'(t) = -x(t) + 4y(t).$$

b. A certain corporation has three branch plants with excess production capacity. All three plants have the capacity for producing a certain product, and management has decided to use some of the excess production capacity as follows. This product can be made in three sizes – large, medium, and small – that yield a net unit profit of \$140, \$120, and \$100, respectively. Plants 1, 2, and 3 have the excess manpower and equipment capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. However, the amount of available in-process storage space also imposes

a limitation on the production rates. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that 900, 1200, and 750 units of the large, medium, and small sizes, respectively, can be sold per day.

To maintain a uniform work load among the plants and to retain some flexibility, management has decided that the additional production assigned to each plant must use the same proportion of the excess manpower and equipment capacity.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

Formulate the linear programming model for this problem [10 marks]

**Question 6**

For each of the following payoff tables, use the graphical procedure described in the lecture to determine the value of the game and the optimal mixed strategy for each player according to the minimax criterion. [20 marks]

(a)

| | | II | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| I | 1 | 4 | 3 | 1 |
| | 2 | 0 | 1 | 2 |

(b)

| | | II | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| I | 1 | 3 | -2 | 4 |
| | 2 | -3 | 6 | -2 |
| | 3 | 1 | -1 | 3 |
| | 4 | 0 | 4 | 1 |