UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER

PROGRAMME: B.SC. IN AGRICULTURAL ECONOMICS AND AGRIBUSINESS MANAGEMENT III (NEW PROGRAMME)

COURSE CODE: AEM 308

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:
1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE CHIEF INVIGILATOR

QUESTION 1

(a) Discuss the possible criteria for fitting a line, stating the advantages and/or disadvantages of each. [10 marks]

(b) The deviations of the observations from the regression line may be attributed to several factors. *List* and *briefly discuss five* (5) of these factors. [10 marks]

(c) Compare *regression analysis* with the *analysis of variance*. [20 marks]

QUESTION 2

The following table gives the quantities of a certain commodity bought in each year from 1961 - 1970 and the corresponding prices.

Year	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**	**1970**
Quantity (Y)	770	785	790	795	800	805	810	820	840	850
Price (X)	18	16	15	15	12	10	10	7	9	6

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 68

Intermediate results

$\sum X = 118$ $\sum Y = 8{,}065$ $\sum X^2 = 1{,}540$ $\sum Y^2 = 6{,}509{,}875$

$\sum XY = 94{,}345$

(a) *Estimate* the *linear demand function* for the commodity. [10 marks]

(b) *Calculate* the *price elasticity* of demand. [10 marks]

(c) *Forecast* the demand at the mean price of the sample. [5 marks]

(d) *Forecast* the demand at *Price* $X = 20$. [5 marks]

QUESTION 3

The following reduced form market equilibrium two-equation simultaneous equations model was used to study the impact of various parameters, including food aid, on the quantity of maize produced and on its official market price in two ecological zones in Swaziland (Lowveld and Lubombo). The first equation in the model captures the effect of food aid and other parameters on quantity of maize produced, while the second depicts the effect of food aid and other parameters on official maize market price.

The main objective was to study the disincentive effect of food aid on domestically produced quantities of maize and its price, but the effect of other variables was also of interest.

$$Q_t = \alpha_o + \alpha_1 P_{t-1} + \alpha_2 w_t + \alpha_3 r_t + \alpha_4 \delta^2_{FA\,t-1} + \alpha_5 I_{t-1} + \alpha_6 FA_{t-1} + D_t + \varepsilon_1$$

$$P_t = \beta_o + \beta_1 Q_{t-1} + \beta_2 w_t + \beta_3 r_t + \beta_4 \delta^2_{FA\,t-1} + \beta_5 I_{t-1} + \beta_6 FA_{t-1} + D_t + \varepsilon_2$$

$$FA_t = \delta_o + \delta_1 Q_{t-1} + \delta_2 w_t + \delta_3 r_t + \delta_4 \delta^2_{FA\,t-1} + \delta_5 I_{t-1} + \delta_6 P_{t-1} + D_t + \varepsilon_3$$

where:

Q = Quantity of maize produced

P = Official (gazzeted) maize price

r = Average fertilizer cost

w = Minimum wage

D = Weather dummy (Drought year: D = 0 if annual amount of precipitation minus 10% or less than the long term annual average rainfall; D = 1 otherwise)

FA = Food aid

$\delta^2_{FA\,t-1}$ = Variance of food aid

I = Commercial cereal imports

α, β and δ are the model coefficients

ε_1 and ε_2 are error terms

t = subscript representing time t to which the value of the variable belongs; $t-1$ if lagged by one year.

Applying the 2-stage least squares method to this model for data for the time period from 1990 to 2006 yielded the following results.

Explanatory Variable	**Quantity of Maize t-value (Probability)**	**Official Price t-value (Probability)**	**Food Aid t-value (Probability)**
Constant	1.36 (0.231)	0.18 (0.860)	0.69 (0.519)
P_t	0.70 (0.518)		0.35 (0.738)
Q_{t-1}		0.83 (0.437)	-1.38 (0.225)
w_t	-1.04 (0.347)	-0.44 (0.677)	-0.14 (0.896)
r_t	0.97 (0.377)	2.57 (0.042)	0.18 (0.868)
$\delta^2_{FA\ t-1}$	-0.14 (0.896)	0.21 (0.840)	0.80 (0.458)
I_{t-1}	-1.91 (0.114)	2.36 (0.056)	0.65 (0.543)
FA_{t-1}	-0.37 (0.730)	-0.39 (0.707)	
D_t	1.78 (0.136)	-3.38 (0.015)	0.42 (0.695)
F-Statistic	2.36 (0.181)	56.870 (0.000)	1.47 (0.346)
R^2	0.77	0.99	0.67
Durbin-Watson Statistic	2.13	2.53	1.46

(a) Comment on the fact that some of the explanatory variables seem to be multicollinear, and suggest ways of combating the problem of multicollinearity. [10 marks]

(b) Comment on the effects of lagging by one year only. What factors could prevent the research from using a model in which variables are lagged by more than one year. [10 marks]

(c) Interpret the results fully, discuss them using your general knowledge of the food security situation in Swaziland, make conclusions and make some policy recommendations. [10 marks]

QUESTION 4

In studying the farm demand for tractors, Griliches used the following model:

$$T_t^* = \alpha X_{1,t-1}^{\beta 1} \; X_{2,t-1}^{\beta 2}$$

where

T^* = desired stock of tractors

X_1 = relative price of tractors

X_2 = interest rate

Using the stock adjustment model, he obtained the following results for the period 1921-1957:

$$\text{Estimated } \log T_t = \text{constant} - 0.218 \log X_{1,t-1} - 0.855 \log X_{2,t-1} - 0.864 \log T_{t-1}$$

$$(0.051) \qquad (0.170) \qquad (0.035)$$

$$R^2 = 0.987$$

where the figures in parentheses are the estimated standard errors.

(a) What is the estimated coefficient of adjustment? [5 marks]
(b) What are the short- and long-run price elasticities? [10 marks]
(c) What are the corresponding interest elasticities? [10 marks]
(d) What are the reasons for high or low rate of adjustment in the present model? [5 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X\sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1\overline{X}$$

$$r^2 = \hat{\beta}_1^2\frac{\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y\sum Y\right)}, \qquad F = \frac{r^2}{1-r^2}(n-2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u{}^2\frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}}}, \qquad \sigma_u{}^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\sigma_u{}^2\frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X\sum X\right)}}}, \qquad \sigma_u{}^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$t = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

$$\text{Total SS} = \sum Y^2 - n\bar{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\bar{Y}^2,$$

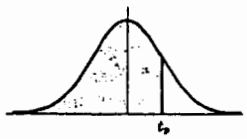
$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n-k-1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n-k-1} = \frac{\text{Total SS - Regression SS}}{n-k-1},$$

$$\hat{\sigma}_{\left(\hat{\beta}_j\right)} = \sqrt{(\text{j}+1)\text{th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0,1,\ldots,k.$$

Appendix III

Percentile Values (t_p)
for
Student's t Distribution
with ν Degrees of Freedom
(shaded area = p)

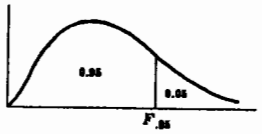


ν	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126

Source: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (5th edition), Table III, Oliver and Boyd Ltd., Edinburgh, by permission of the authors and publishers.

Appendix V

95th Percentile Values for the *F* Distribution
(ν_1 degrees of freedom in numerator)
(ν_2 degrees of freedom in denominstor)

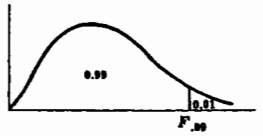


ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 178, by permission.

Appendix VI

99th Percentile Values for the *F* Distribution
(ν_1 degrees of freedom in numerator)
(ν_2 degrees of freedom in denominator)



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	5000	5403	5625	5764	5859	5928	5981	6023	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.16	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 180, by permission.