UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER

PROGRAMME: B.SC. AGRICULTURE (AGRICULTURAL ECONOMICS AND MANAGEMENT OPTION) YEAR IV (OLD PROGRAMME)

COURSE CODE: AEM 401

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

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QUESTION 1

(a) *State* and *briefly discuss* the assumptions of the simple linear regression model. [20 marks]

(b) *Give* and *briefly discuss* seven (7) desirable *small sample properties* of estimators. [20 marks]

QUESTION 2

A random sample of ten (10) families had the following income and food expenditure (in £ per week):

Families	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Family Income (X)	20	30	33	40	15	13	26	38	35	43
Family Expenditure (Y)	7	9	8	11	5	4	8	10	9	10

Source: Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 67

Intermediate results

$\sum X = 293$ $\sum Y = 81$ $\sum X^2 = 9{,}577$ $\sum Y^2 = 701$

$\sum XY = 2{,}574$

(a) Estimate the regression line of *food expenditure* on *income*. [10 marks]

(b) Calculate the coefficient of determination, R^2. Conduct a test of significance of this coefficient at the 5% level of significance. Provide an economic interpretation for the results of your test. [**N.B.**: You need not construct the relevant table for the test.] [10 marks]

(c) Calculate the standard errors of the estimated parameters and conduct tests of significance using the *standard error test*. If appropriate, give an *economic interpretation* for the *regression coefficient*. [10 marks]

QUESTION 3

The following table shows the price index of durables, the average yearly income and expenditure on durables of a 'typical' household in country *Z*.

Year	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
Expenditure on Durables Y (£)	115	110	115	120	140	100	105	95	135	105
Household Income X_1 (£)	1855	2000	2010	2040	2275	2255	1995	1905	2355	2035
Price index X_2	100	102	95	95	94	110	110	112	115	120

Source: Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 173

Intermediate results:

$\sum Y = 1,140$ $\sum X_1 = 20,725$ $\sum X_2 = 1,053$

$\sum X_1Y = 2,375,325$ $\sum X_1^2 = 43,199,675$ $\sum X_1X_2 = 2,183,985$

$\sum X_2Y = 119,520$ $\sum Y^2 = 131,850$ $\sum X_2^2 = 111,659$

(a) Fit a regression plane to the function

$$Y = \beta_o + \beta_1X_1 + \beta_2X_2 + U.$$ [10 marks]

(b) Test the overall significance of the regression. [10 marks]

(c) If appropriate, give the economic meaning of the statistical significance of the partial regression coefficients for Household Income (X_1) and Price Index (X_2). [10 marks]

QUESTION 4

The following results were obtained from a sample of 15 firms of the chemical industry on their output (Y), labour input (X_1) and capital input (X_2), measured in arbitrary units.

$\sum Y^* = 86.159$ $\sum Y^{*2} = 500.181$ $\sum Y^*X_1^* = 638.339$ $\sum X_1^* = 110.885$
$\sum X_1^{*2} = 820.118$ $\sum Y^*X_2^* = 553.851$ $\sum X_2^* = 95.966$ $\sum X_2^{*2} = 615.701$
$\sum X_1^*X_2^* = 710.138$

Here,

$Y^* = \log_e Y$
$X_1^* = \log_e X_1$

159

$X_2^* = \log_e X_2$

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P.139

(a) Fit a Cobb-Douglas production function using the above results

$$Y = \beta_o X_1^{\beta 1} X_2^{\beta 2} e^u.$$ [10 marks]

(b) Conduct tests of significance of the intercept and the constat elasticities and provide their economic meaning. [20 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X \sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1 \overline{X}$$

$$r^2 = \hat{\beta}_1^{\,2} \frac{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y \sum Y\right)}, \qquad F = \frac{r^2}{1 - r^2}(n - 2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\sigma_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}}, \qquad \sigma_u^2 \text{ known}$$

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$$t = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^{\,2} \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^{\,2} \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

$$\text{Total SS} = \sum Y^2 - n\bar{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\bar{Y}^2,$$

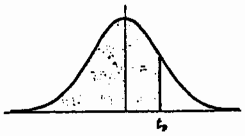
$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n-k-1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n-k-1} = \frac{\text{Total SS - Regression SS}}{n-k-1},$$

$$\hat{\sigma}_{(\hat{\beta}_j)} = \sqrt{(j+1)\text{th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0,1,\ldots,k.$$

Appendix III

Percentile Values (t_p)
for
Student's t Distribution
with ν Degrees of Freedom
(shaded area = p)

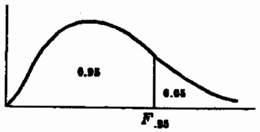


ν	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126

Source: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (5th edition), Table III, Oliver and Boyd Ltd., Edinburgh, by permission of the authors and publishers.

Appendix V

95th Percentile Values for the *F* Distribution
(ν_1 degrees of freedom in numerator)
(ν_2 degrees of freedom in denominator)

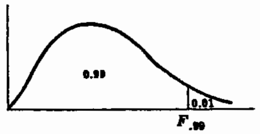


ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 178, by permission.

Appendix VI

99th Percentile Values for the *F* Distribution
(ν_1 degrees of freedom in numerator)
(ν_2 degrees of freedom in denominator)



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	5000	5403	5625	5764	5859	5928	5981	6023	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 180, by permission.