UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER

PROGRAMME: B.SC. AGRICULTURAL ECONOMICS AND AGRIBUSINESS MANAGEMENT YEAR III (NEW PROGRAMME)

COURSE CODE: AEM 308

TITLE OF PAPER: INTRODUCTION TO ECONOMETRICS

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION:

1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS.
2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.

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QUESTION 1

(a) *List* and *briefly discuss* the *small sample properties of estimators.* [14 marks]

(b) State the *Gauss-Markov* theorem and give its meaning [6 marks]

(c) The quantity supplied of a commodity X is assumed to be a linear function of the price of x and the wage rate of labour used in the production of x. The population supply equation is given as

$$Q = \beta_o + \beta_1 P_x + \beta_2 W + U$$

where Q = quantity supplied of x
P_x = price of x
W = wage rate
U = random error term

The data in the table below were obtained from a sample of the population.

Y=Q	20	35	30	47	60	68	76	90	100	105	130	140	125	120	135
X_1=P_x	10	15	21	26	40	37	42	33	30	38	60	65	50	35	42
X_2=W	12	10	9	8	5	7	4	5	7	5	3	4	3	1	2

Source:Koutsoyiannis, A., 2/Ed, 1981. *Theory of Econometrics*. P. 606

Intermediate results:

$\sum Y = 1,281$ $\sum X_1 = 544$ $\sum X_2 = 85$

$\sum X_1 Y = 53,665$ $\sum X_1^2 = 22,922$ $\sum X_1 X_2 = 2,568$

$\sum X_2 Y = 5,706$ $\sum Y^2 = 132,609$ $\sum X_2^2 = 617$

The ANOVA table for the regression analysis was as as given below:

```
*** Summary of analysis ***

              d.f.        s.s.        m.s.      v.r.   F pr.
Regression       2      19192.      9595.9     28.65   <.001
Residual        12       4020.       335.0
Total           14      23212.      1658.0

Percentage variance accounted for 79.8
```

What general conclusion can you draw from this output? [4 marks]

The estimates of parameters were as given below.

```
*** Estimates of parameters ***

                  estimate          s.e.     t(12)  t pr.
Constant              89.5          31.7      2.82  0.015
Price                1.053         0.521      2.02  0.066
Wage_rate            -7.47          2.53     -2.95  0.012
```

What specific conclusions can you draw from this output? [16 marks]

Further output gave the following plots of the residuals and the Durbin Watson Statistic was determined to be 1.181.

QUESTION 2

In studying the purchase of durable goods Y ($Y = 1$ if purchased, $Y = 0$ if no purchase) as a function of several variables for a total of 762 households, Janet A. Fisher obtained the following linear probability model (LPM) results:

Explanatory variable	Coefficient	Standard error
Constant	0.1411	–
1957 disposable income, X_1	0.0251	0.0118
(Disposable income $= X_1)^2$, X_2	−0.0004	0.0004
Checking accounts, X_3	−0.0051	0.0108
Savings accounts, X_4	0.0013	0.0047
U.S. Savings Bonds, X_5	−0.0079	0.0067
Housing status: rent, X_6	−0.0469	0.0937
Housing status: own, X_7	0.0136	0.0712
Monthly rent, X_8	−0.7540	1.0983
Monthly mortgage repayments, X_9	−0.9809	0.5162
Personal noninstallment debt, X_{10}	−0.0367	0.0326
Age, X_{11}	0.0046	0.0084
Age squared, X_{12}	−0.0001	0.0001
Marital status, X_{13} (1 = married)	0.1760	0.0501
Number of children, X_{14}	0.0398	0.0358
(Number of children $= X_{14})^2$, X_{15}	−0.0036	0.0072
Purchase plans, X_{16} (1 = planned; 0 otherwise)	0.1760	0.0384
	$R^2 = 0.1336$	

Notes: All financial variables are in thousands of dollars.
Housing status: Rent (1 if rents; 0 otherwise)
Housing status Own (1 if owns; 0 otherwise)

Source: Janet A. Fisher, "An Analysis of Consumer Good Expenditure," *The Review of Economics and Statistics*, Vol. 64, no.1, Table 1, 1962, p. 67.

(a) Comment generally on the fit of the equation. [6 marks]

(b) How would you interpret the coefficient of −0.0051 attached to the checking account variable? How would you rationalize the negative sign for this variable? [6 marks]

(c) What is the rationale behind introducing the age-squared and the number of children-squared variables? Why is the sign negative in both cases? [6 marks]

(d) Assuming values of zero for all except the income variable, find out the conditional probability that a household with income of $20,000 will purchase a durable good. [6 marks]

(e) Estimate the conditional probability of owning durable good(s), given: X_1 = $15,000, X_3 = $3,000, X_4 = $5,000, $X_6 = 0$, $X_7 = 1$, X_8 = $500, X_9 = $300, $X_{10} = 0$, $X_{11} = 35$, $X_{13} = 1$, $X_{14} = 2$, $X_{16} = 0$. [6 marks]

QUESTION 3

From the model:

$$Y_{1t} = \beta_{10} + \beta_{12}Y_{2t} + \gamma_{11}X_{1t} + u_{1t}$$

$$Y_{2t} = \beta_{20} + \beta_{21}Y_{1t} + u_{2t}$$

the following reduced-form equations are obtained

$$Y_{1t} = \pi_{10} + \pi_{11}X_{1t} + w_t$$

$$Y_{2t} = \pi_{20} + \pi_{21}X_{1t} + v_t$$

(a) Express the reduced-form parameters (π's) in terms of the structural parameters (β's and γ's). [10 marks]

(b) Are the structural equations identified? [2 marks]

(c) What happens to identification if it is known a priori that (1) $\beta_{12} = 0$ and (2) $\beta_{10} = 0$? [3 marks]

(d) Suppose that the estimated reduced-form equations are as follows:

$$\text{Estimated } Y_{1t} = 4 + 8X_{1t}$$

$$\text{Estimated } Y_{2t} = 2 + 12X_{1t}$$

(i) Obtain the values of the structural parameters, where possible. [10 marks]

(ii) How would you test the null hypothesis that $\gamma_{11} = 0$? [5 marks]

QUESTION 4

From the quarterly data for the period 1950-1960, F.P.R. Brechling obtained the following demand function for labour for British economy (the figures in parentheses are standard errors):

$$\hat{E}_{d,t} = \underset{(2.61)}{14.22} + \underset{(0.014)}{0.172Q_t} - \underset{(0.015)}{0.028t} - \underset{(0.0002)}{0.0007t^2} - \underset{(0.033)}{0.297E_{t-1}}$$

$$\text{Adj. } R^2 = 0.76 \qquad d = 1.37$$

where

$E_{d,t} = (E_t - E_{t-1})$

Q = output

t = time

The preceding equation was based on the assumption that the desired level of employment E_t^* is a function of output, time, and time squared and the hypothesis that $E_t - E_{t-1} = \delta(E_t^* - E_{t-1})$, where δ, the coefficient of adjustment, lies between 0 and 1.

(a) Interpret the preceding regression. [10 marks]

(b) What is the value of δ? [5 marks]

(c) Derive the long-run demand function for labour from the estimated short-run demand function. [10 marks]

(d) How would you test for serial correlation in the preceding model? [5 marks]

FORMULAE

$$\hat{\beta}_1 = \frac{\left(\sum XY - \frac{1}{n}\sum X \sum Y\right)}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}, \qquad \hat{\beta}_o = \overline{Y} - \hat{\beta}_1 \overline{X}$$

$$r^2 = \hat{\beta}_1^2 \frac{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}{\left(\sum Y^2 - \frac{1}{n}\sum Y \sum Y\right)}, \qquad F = \frac{r^2}{1 - r^2}(n-2)$$

$$Z = \frac{\hat{\beta}_o}{\sqrt{\sigma_u^2 \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$

σ_u^2 known

$$Z = \frac{\hat{\beta}_1}{\sqrt{\sigma_u^2 \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$

σ_u^2 known

$$Z = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^2 \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$Z = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^2 \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n>30$

$$t = \frac{\hat{\beta}_o}{\sqrt{\hat{\sigma}_u^2 \frac{\sum X^2}{n\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$t = \frac{\hat{\beta}_1}{\sqrt{\hat{\sigma}_u^2 \frac{1}{\left(\sum X^2 - \frac{1}{n}\sum X \sum X\right)}}},$$ σ_u^2 is unknown and $n\leq 30$

$$\hat{\eta} = \hat{\beta}_1 \frac{\overline{X}}{\overline{Y}}$$

FORMULAE (IN MATRIX FORM)

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y,$$

$$X^T X = \begin{pmatrix} n & \sum X \\ \sum X & \sum X^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum XY \end{pmatrix},$$

$$X^T X = \begin{pmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{pmatrix}, \qquad X^T Y = \begin{pmatrix} \sum Y \\ \sum X_1 Y \\ \sum X_2 Y \end{pmatrix},$$

$$\left(X^T X\right)^{-1} = \frac{1}{\det\left(X^T X\right)} \operatorname{cof}\left(X^T X\right),$$

$$\text{Total SS} = \sum Y^2 - n\overline{Y}^2, \qquad \text{Regression SS} = \hat{\beta}^T X^T Y - n\overline{Y}^2,$$

$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}}, \qquad F = \frac{R^2}{1 - R^2} \cdot \frac{n - k - 1}{k},$$

$$\hat{\sigma}_u^2 = \frac{\text{Error SS}}{n - k - 1} = \frac{\text{Total SS} - \text{Regression SS}}{n - k - 1},$$

$$\hat{\sigma}_{(\hat{\beta}_j)} = \sqrt{(j+1)\text{th entry of } diag\left[\hat{\sigma}_u^2 \left(X^T X\right)^{-1}\right]}, \qquad \text{where } j = 0, 1, \ldots, k.$$

Appendix E Points for the Distribution of F [5% (light type) and 1% (bold face type)]

f_2	f_1 Degrees of freedom (for greater mean square)																								f_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
1	161 **4,052**	200 **4,999**	216 **5,403**	225 **5,625**	230 **5,764**	234 **5,859**	237 **5,928**	239 **5,981**	241 **6,022**	242 **6,056**	243 **6,082**	244 **6,106**	245 **6,142**	246 **6,169**	248 **6,208**	249 **6,234**	250 **6,261**	251 **6,286**	252 **6,302**	253 **6,323**	253 **6,334**	254 **6,352**	254 **6,361**	254 **6,366**	1
2	18.51 **98.49**	19.00 **99.00**	19.16 **99.17**	19.25 **99.25**	19.30 **99.30**	19.33 **99.33**	19.36 **99.36**	19.37 **99.37**	19.38 **99.39**	19.39 **99.40**	19.40 **99.41**	19.41 **99.42**	19.42 **99.43**	19.43 **99.44**	19.44 **99.45**	19.45 **99.46**	19.46 **99.47**	19.47 **99.48**	19.47 **99.48**	19.48 **99.49**	19.49 **99.49**	19.49 **99.49**	19.50 **99.50**	19.50 **99.50**	2
3	10.13 **34.12**	9.55 **30.82**	9.28 **29.46**	9.12 **28.71**	9.01 **28.24**	8.94 **27.91**	8.88 **27.67**	8.84 **27.49**	8.81 **27.34**	8.78 **27.23**	8.76 **27.13**	8.74 **27.05**	8.71 **26.92**	8.69 **26.83**	8.66 **26.69**	8.64 **26.60**	8.62 **26.50**	8.60 **26.41**	8.58 **26.35**	8.57 **26.27**	8.56 **26.23**	8.54 **26.18**	8.54 **26.14**	8.53 **26.12**	3
4	7.71 **21.20**	6.94 **18.00**	6.59 **16.69**	6.39 **15.98**	6.26 **15.52**	6.16 **15.21**	6.09 **14.98**	6.04 **14.80**	6.00 **14.66**	5.96 **14.54**	5.93 **14.45**	5.91 **14.37**	5.87 **14.24**	5.84 **14.15**	5.80 **14.02**	5.77 **13.93**	5.74 **13.83**	5.71 **13.74**	5.70 **13.69**	5.68 **13.61**	5.66 **13.57**	5.65 **13.52**	5.64 **13.48**	5.63 **13.46**	4
5	6.61 **16.26**	5.79 **13.27**	5.41 **12.06**	5.19 **11.39**	5.05 **10.97**	4.95 **10.67**	4.88 **10.45**	4.82 **10.29**	4.78 **10.15**	4.74 **10.05**	4.70 **9.96**	4.68 **9.89**	4.64 **9.77**	4.60 **9.68**	4.56 **9.55**	4.53 **9.47**	4.50 **9.38**	4.46 **9.29**	4.44 **9.24**	4.42 **9.17**	4.40 **9.13**	4.38 **9.07**	4.37 **9.04**	4.36 **9.02**	5
6	5.99 **13.74**	5.14 **10.92**	4.76 **9.78**	4.53 **9.15**	4.39 **8.75**	4.28 **8.47**	4.21 **8.26**	4.15 **8.10**	4.10 **7.98**	4.06 **7.87**	4.03 **7.79**	4.00 **7.72**	3.96 **7.60**	3.92 **7.52**	3.87 **7.39**	3.84 **7.31**	3.81 **7.23**	3.77 **7.14**	3.75 **7.09**	3.72 **7.02**	3.71 **6.99**	3.69 **6.94**	3.68 **6.90**	3.67 **6.88**	6
7	5.59 **12.25**	4.74 **9.55**	4.35 **8.45**	4.12 **7.85**	3.97 **7.46**	3.87 **7.19**	3.79 **7.00**	3.73 **6.84**	3.68 **6.71**	3.63 **6.62**	3.60 **6.54**	3.57 **6.47**	3.52 **6.35**	3.49 **6.27**	3.44 **6.15**	3.41 **6.07**	3.38 **5.98**	3.34 **5.90**	3.32 **5.85**	3.29 **5.78**	3.28 **5.75**	3.25 **5.70**	3.24 **5.67**	3.23 **5.65**	7
8	5.32 **11.26**	4.46 **8.65**	4.07 **7.59**	3.84 **7.01**	3.69 **6.63**	3.58 **6.37**	3.50 **6.19**	3.44 **6.03**	3.39 **5.91**	3.34 **5.82**	3.31 **5.74**	3.28 **5.67**	3.23 **5.56**	3.20 **5.48**	3.15 **5.36**	3.12 **5.28**	3.08 **5.20**	3.05 **5.11**	3.03 **5.06**	3.00 **5.00**	2.98 **4.96**	2.96 **4.91**	2.94 **4.88**	2.93 **4.86**	8
9	5.12 **10.56**	4.26 **8.02**	3.86 **6.99**	3.63 **6.42**	3.48 **6.06**	3.37 **5.80**	3.29 **5.62**	3.23 **5.47**	3.18 **5.35**	3.13 **5.26**	3.10 **5.18**	3.07 **5.11**	3.02 **5.00**	2.98 **4.92**	2.93 **4.80**	2.90 **4.73**	2.86 **4.64**	2.82 **4.56**	2.80 **4.51**	2.77 **4.45**	2.76 **4.41**	2.73 **4.36**	2.72 **4.33**	2.71 **4.31**	9
10	4.96 **10.04**	4.10 **7.56**	3.71 **6.55**	3.48 **5.99**	3.33 **5.64**	3.22 **5.39**	3.14 **5.21**	3.07 **5.06**	3.02 **4.95**	2.97 **4.85**	2.94 **4.78**	2.91 **4.71**	2.86 **4.60**	2.82 **4.52**	2.77 **4.41**	2.74 **4.33**	2.70 **4.25**	2.67 **4.17**	2.64 **4.12**	2.61 **4.05**	2.59 **4.01**	2.56 **3.96**	2.55 **3.93**	2.54 **3.91**	10
11	4.84 **9.65**	3.98 **7.20**	3.59 **6.22**	3.36 **5.67**	3.20 **5.32**	3.09 **5.07**	3.01 **4.88**	2.95 **4.74**	2.90 **4.63**	2.86 **4.54**	2.82 **4.46**	2.79 **4.40**	2.74 **4.29**	2.70 **4.21**	2.65 **4.10**	2.61 **4.02**	2.57 **3.94**	2.53 **3.86**	2.50 **3.80**	2.47 **3.74**	2.45 **3.70**	2.42 **3.66**	2.41 **3.62**	2.40 **3.60**	11
12	4.75 **9.33**	3.88 **6.93**	3.49 **5.95**	3.26 **5.41**	3.11 **5.06**	3.00 **4.82**	2.92 **4.65**	2.85 **4.50**	2.80 **4.39**	2.76 **4.30**	2.72 **4.22**	2.69 **4.16**	2.64 **4.05**	2.60 **3.98**	2.54 **3.86**	2.50 **3.78**	2.46 **3.70**	2.42 **3.61**	2.40 **3.56**	2.36 **3.49**	2.35 **3.46**	2.32 **3.41**	2.31 **3.38**	2.30 **3.36**	12
13	4.67 **9.07**	3.80 **6.70**	3.41 **5.74**	3.18 **5.20**	3.02 **4.86**	2.92 **4.62**	2.84 **4.44**	2.77 **4.30**	2.72 **4.19**	2.67 **4.10**	2.63 **4.02**	2.60 **3.96**	2.55 **3.85**	2.51 **3.78**	2.46 **3.67**	2.42 **3.59**	2.38 **3.51**	2.34 **3.42**	2.32 **3.37**	2.28 **3.30**	2.26 **3.27**	2.24 **3.21**	2.22 **3.18**	2.21 **3.16**	13

continued next page

Appendix C Distribution of *t* Probability

n	.9	.8	.7	.6	.5	.4	.3	.2	.1	.05	.02	.01	.001
1	.158	.325	.510	.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	.142	.289	.445	.617	.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	.137	.277	.424	.584	.765	.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	.134	.271	.414	.569	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	.132	.267	.408	.559	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	.131	.265	.404	.553	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	.130	.263	.402	.549	.711	.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	.130	.262	.399	.546	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	.129	.261	.398	.543	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	.129	.260	.396	.540	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	.128	.259	.395	.539	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	.128	.259	.394	.538	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	.128	.258	.393	.537	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	.128	.258	.393	.536	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	.128	.258	.392	.535	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	.128	.257	.392	.534	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	.127	.257	.392	.534	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	.127	.257	.391	.533	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	.127	.257	.391	.533	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	.127	.257	.391	.532	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	.127	.256	.390	.532	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	.127	.256	.390	.531	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	.127	.256	.390	.531	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	.127	.256	.390	.531	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	.127	.256	.389	.531	.684	.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	.127	.256	.389	.530	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	.127	.256	.389	.530	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	.127	.256	.389	.530	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	.126	.255	.388	.529	.681	.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	.126	.254	.387	.527	.679	.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	.126	.254	.386	.526	.677	.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	.126	.253	.385	.524	.674	.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

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