UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2017

TITLE OF PAPER	:	BUSINESS QUANTITATIVE METHODS
COURSE CODE	:	BUS611
TIME ALLOWED	:	THREE (3) HOURS
REQUIREMENTS	:	CALCULATOR
INSTRUCTIONS	:	ANSWER ANY FOUR (4) QUESTIONS

Question 1 [25 marks, 3+2+4+4+4+4+4]

(a) A survey was conducted by UNISWA students to find out the monthly consumption of water per household in a city during the recent summer: The data below shows consumption of water from 13 households:

Cubic metres per month:

342	426	317	545	264	451	1049
631	512	266	492	562	298	

(i) Compute the mean, median, and mode.

(ii) Looking at the distribution of the spending, which measures of location do you think are best and/or worst? Why?

(iii) Compute the standard deviation.

(b) The manager of a shop (store **AB**) wants to study characteristics of customers. In particular he decides to focus on two variables: the amount of money spent by customers on clothes and whether the customers have one child, two children or more than two children. The results from a sample of 70 customers are as follows:

Amount of money spent: $\bar{x}$ = SZL 213.40, s = SZL 92.20
37 customers have only one child
26 customers have two children
7 customers have more than two children

(i) Set up a 95% confidence interval estimate of the population mean amount spent at the shop.

(ii) Set up a 90% confidence interval estimate of the population proportion of customers who have two children.

If the manager of another shop wants to conduct a similar survey and does not have access to the information generated by the manager from the store **AB**.

(iii) If he wants to have 95% confidence of estimating the true population mean amount spent in his store to within ±SZL 15.00 and the standard deviation is assumed to be within SZL 100, what sample size is needed?

(iv) If he wants to have 90% confidence of estimating the population proportion of customers who have two kids to be within ±0.045, what sample size is needed?

Question 2 [25 marks, 2+7+2+6+2+2+2+2]

(a) The following relates to the number of tourist arrivals in thousands (Y) in Swaziland from the UK and the gross domestic product of Swaziland in SZL billion (X), reflecting the level of economic development over the period 1986 to 2000.

Number of observation = 15
$\sum y = 756$
$\sum x = 108$
$\sum y^2 = 48,522$
$\sum x^2 = 1,020$
$\sum xy = 6,960$

(i) Assuming a linear relationship as follows

$$Y = \beta_0 + \beta_1 X$$

What is the expected sign of β_1? Why?

(ii) Use the least-squares method to find the regression coefficients, β_0 and β_1.

(iii) Interpret the meaning of the slope coefficient.

(iv) Determine the coefficient of determination, r^2, and interpret its meaning.

(v) From (iv), what is the correlation coefficient?

(vi) Predict the number of tourist arriving in Swaziland when the level of income in Swaziland is SZL 15 billion.

(b) A nightclub obtains the following data on the age and marital status of 140 customers.

	Marital status	
	Single	Married
Under 25	77	14
25 or over	28	21

(i) What is the probability a customer is married and under 25?

(ii) If a customer is under 25, what is the probability that he or she is single?

(iii) Is marital status independent of age? Explain your answer.

Question 3 [25 marks, 3+3+3+3+13]

The following payoff table indicates profit (in SZL) for a particular product, which will result from each of three alternative decisions assuming either high or low demand of the product. You may assume there is an 80 % chance of high demand and a 20% chance of low demand.

	High Demand	Low Demand
Decision 1	150,000	-50,000
Decision 2	120,000	20,000
Decision 3	80,000	50,000

Briefly describe what is meant by each of the following decision-making criteria:

(a) The maximax criterion

(b) The maximin criterion

(c) The minimax regret criterion

(d) The expected value criterion

Using each of the criteria above, determine which decision should be chosen. Show your workings and explain your answer in each case.

Question 4 [25 marks, 10+6+9]

A furniture manufacturer produces two types of desks: Standard and Executive. These desks are then sold at SZL3000 for the Standard type and SZL3300 for the Executive type to an office furniture wholesaler; there is an unlimited market for any mix of these desks, atleast within the manufacturers production capacity. Each desk has to go through four basic operations: cutting of the timber, joining of the pieces, pre-finishing and final finish. Each unit of the Standard desk produced takes 48 minutes of cutting time, 2 hours of joining, 40 minutes of pre-finishing and 4 hours of final finishing time. Each unit of the Executive desk required 72 minutes of cutting, 3 hours of joining, 2 hours of prefinishing and 5 hours and 20 minutes of final finishing time. The daily capacity for each operation amounts to 16 hours of cutting, 30 hours of joining, 16 hours of pre-finishing and 64 hours of final finishing time. It costs the furniture manufacturer SZL2600 and SZL2800 to produce one unit of Standard desk and one unit of Executive desk respectively.

(a) Formulate the linear programming model for this problem.

(b) Plot a graph indicating and labelling clearly all the constraints, the feasible region and the corner points for the LP problem.

(c) Determine the product mix that will maximise the total revenue using the corner point method.

Question 5 [25 marks, 4+2+8+2+3+2+2]

(a) It is claimed that the amount donated to charity varies by district. To investigate this you find data from a random sample of individuals in Hhohho and compare this to a random sample of individuals from Manzini. The amounts donated per month (in SZL) are recorded in the following table:

	Hhohho	Manzini
Sample Size	24	21
Mean	97.42	201.19
Standard Deviation	115.546	205.645

Let μ_1 represent the population mean monthly donations for Hhohho and μ_2 the population mean monthly donations for Manzini.

(i) Is there evidence that the mean monthly donations in Manzini is greater than SZL 195? Perform an appropriate test (use the 5% significance level).

(ii) Combine the two sample standard deviations to obtain a "pooled" sample standard deviation, Sp.

(iii) Does a comparison of the two samples reveal individuals living in Manzini donate more compare to individuals living in the Hhohho region? To answer this conduct a 2-sample t-test (use the 5% significance level).

(b) A report by Grant Thornton suggests the mean basic salary for bosses of the largest UK companies (FTSE 100 Executives) was 583,291 in 2014. Assume the standard deviation was 451,151. Assume the population is normally distributed.

(i) What is the probability that a randomly selected boss has a salary between 1 million and 1.5 million?

(ii) Ten percent of bosses have a salary of how much or less?

You obtain a random sample of basic salary for 5 FTSE 100 Executives.

(iii) Find the standard error of the sample mean salary.

(iv) What is the probability that the sample mean is greater than 620,000?

(v) What is the probability that the sample mean differs from the population mean by more than 100,000?

Question 6 [25 marks, 2+4+6+3+2+4+4]

The following table gives details of a garage building project. It lists all activities involved together with any immediately preceding activities and the time in days required to complete each activity.

Activity	Description	Immediately Preceding Acivities	Duration(days)
A	Obtain bricklayer	-	10
B	Dig the foundations	-	8
C	Lay the base	B	1
D	Build the walls	A,C	8
E	Build the roof	D	3
F	Tile the roof	E	2
G	Make window frames	-	3
H	Fit the window frames	D,G	1
I	Fit glass to frames	H	1
J	Fit the door	E	1
K	Paint doors/windows	I,J	2
L	Point the brickwork	D	2

(a) By looking at the precedence table can you identify whether or not a dummy activity will be necessary when drawing the network. Explain.

(b) Draw a network for the garage building project.

(c) Calculate the earliest event time and latest event time for each node.

(d) Find the shortest total duration for the project.

(e) Determine the float for each activity.

(f) Identify the critical path(s) through the network.

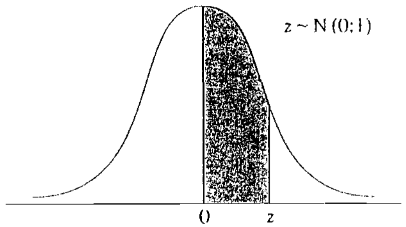
(g) Briefly describe what is meant by a 'Gantt Chart' and explain its uses in relation to a project such as the one above.

APPENDIX 1: LIST OF STATISTICAL TABLES

TABLE 1

The standard normal distribution (z)

This table gives the area under the standard normal curve between 0 and z
i.e. $P[0 < Z < z]$

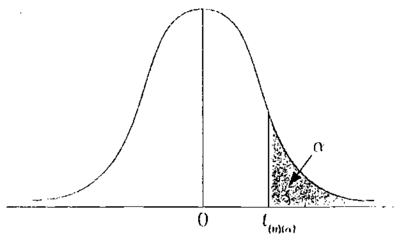


Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2703	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49897	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49991	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

TABLE 2

The t distribution

This table gives the value of $t_{(n)(\alpha)}$ where n is the degrees of freedom
i.e. $\alpha = P[t \geq t_{(n)(\alpha)}]$

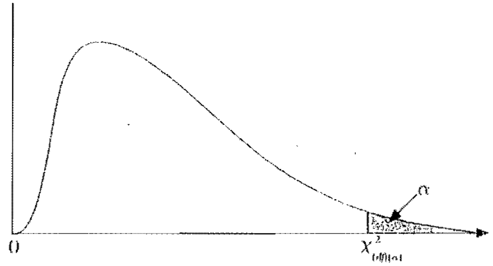


α / df	0.100	0.050	0.025	0.010	0.005	0.0025
1	3.078	6.314	12.706	31.821	63.657	127.322
2	1.886	2.920	4.303	6.965	9.925	14.089
3	1.638	2.353	3.182	4.541	5.841	7.453
4	1.533	2.132	2.776	3.747	4.604	5.598
5	1.476	2.015	2.571	3.365	4.032	4.773
6	1.440	1.943	2.447	3.143	3.707	4.317
7	1.415	1.895	2.365	2.998	3.499	4.029
8	1.397	1.860	2.306	2.896	3.355	3.833
9	1.383	1.833	2.262	2.821	3.250	3.690
10	1.372	1.812	2.228	2.764	3.169	3.581
11	1.363	1.796	2.201	2.718	3.106	3.497
12	1.356	1.782	2.179	2.681	3.055	3.428
13	1.350	1.771	2.160	2.650	3.012	3.372
14	1.345	1.761	2.145	2.624	2.977	3.326
15	1.341	1.753	2.131	2.602	2.947	3.286
16	1.337	1.746	2.120	2.583	2.921	3.252
17	1.333	1.740	2.110	2.567	2.898	3.222
18	1.330	1.734	2.101	2.552	2.878	3.197
19	1.328	1.729	2.093	2.539	2.861	3.174
20	1.325	1.725	2.086	2.528	2.845	3.153
21	1.323	1.721	2.080	2.518	2.831	3.135
22	1.321	1.717	2.074	2.508	2.819	3.119
23	1.319	1.714	2.069	2.500	2.807	3.104
24	1.318	1.711	2.064	2.492	2.797	3.091
25	1.316	1.708	2.060	2.485	2.787	3.078
26	1.315	1.706	2.056	2.479	2.779	3.067
27	1.314	1.703	2.052	2.473	2.771	3.057
28	1.313	1.701	2.048	2.467	2.763	3.047
29	1.311	1.699	2.045	2.462	2.756	3.038
30	1.310	1.697	2.042	2.457	2.750	3.030
31	1.309	1.696	2.040	2.453	2.744	3.022
32	1.309	1.694	2.037	2.449	2.738	3.015
33	1.308	1.692	2.035	2.445	2.733	3.008
34	1.307	1.691	2.032	2.441	2.728	3.002
35	1.306	1.690	2.030	2.438	2.724	2.996
36	1.306	1.688	2.028	2.434	2.719	2.990
37	1.305	1.687	2.026	2.431	2.715	2.985
38	1.304	1.686	2.024	2.429	2.712	2.980
39	1.304	1.685	2.023	2.426	2.708	2.976
40	1.303	1.684	2.021	2.423	2.704	2.971
45	1.301	1.679	2.014	2.412	2.690	2.952
50	1.299	1.676	2.009	2.403	2.678	2.937
60	1.296	1.671	2.000	2.390	2.660	2.915
70	1.294	1.667	1.994	2.381	2.648	2.899
80	1.292	1.664	1.990	2.374	2.639	2.887
90	1.291	1.662	1.987	2.369	2.632	2.878
100	1.290	1.660	1.984	2.364	2.626	2.871
110	1.289	1.659	1.982	2.361	2.621	2.865
120	1.289	1.658	1.980	2.358	2.617	2.860
140	1.288	1.656	1.977	2.353	2.611	2.852
160	1.287	1.654	1.975	2.350	2.607	2.847
180	1.286	1.653	1.973	2.347	2.603	2.842
200	1.286	1.653	1.972	2.345	2.601	2.839
[illegible]	[illegible]	1.645	1.960	[illegible]	[illegible]	2.807

TABLE 3

The Chi-Squared distribution (χ^2)

This table gives the value of $\chi^2_{(df)(\alpha)}$ where df is the degrees of freedom i.e. $\alpha = P[\chi^2 > \chi^2_{(df)(\alpha)}]$

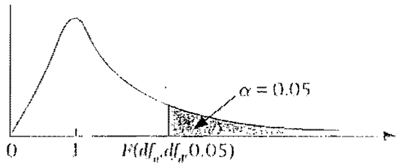


df \ α	0.100	0.050	0.025	0.01	0.005	0.0025
1	2.707	3.843	5.026	6.637	7.881	9.142
2	4.605	5.991	7.378	9.210	10.597	11.983
3	6.251	7.815	9.348	11.345	12.838	14.320
4	7.779	9.488	11.143	13.277	14.860	16.424
5	9.236	11.070	12.833	15.086	16.750	18.386
6	10.645	12.592	14.449	16.812	18.548	20.249
7	12.017	14.067	16.013	18.478	20.278	22.040
8	13.362	15.507	17.535	20.090	21.955	23.774
9	14.684	16.919	19.023	21.666	23.589	25.462
10	15.987	18.307	20.483	23.209	25.188	27.112
11	17.275	19.675	21.920	24.725	26.757	28.729
12	18.549	21.026	23.337	26.217	28.300	30.318
13	19.812	22.362	24.736	27.688	29.819	31.883
14	21.064	23.685	26.119	29.141	31.319	33.426
15	22.307	24.996	27.488	30.578	32.801	34.950
16	23.542	26.296	28.845	32.000	34.267	36.456
17	24.769	27.587	30.191	33.409	35.718	37.946
18	25.989	28.869	31.526	34.805	37.156	39.422
19	27.204	30.144	32.852	36.191	38.582	40.885
20	28.412	31.410	34.170	37.566	39.997	42.336
21	29.615	32.671	35.479	38.932	41.401	43.775
22	30.813	33.924	36.781	40.289	42.796	45.204
23	32.007	35.172	38.076	41.638	44.181	46.623
24	33.196	36.415	39.364	42.980	45.558	48.034
25	34.382	37.652	40.646	44.314	46.928	49.435
26	35.563	38.885	41.923	45.642	48.290	50.829
27	36.741	40.113	43.195	46.963	49.645	52.215
28	37.916	41.337	44.461	48.278	50.993	53.594
29	39.087	42.557	45.722	49.588	52.336	54.967
30	40.256	43.773	46.979	50.892	53.672	56.332
31	44.422	44.985	48.232	52.191	55.003	57.692
32	42.585	46.194	49.480	53.486	56.328	59.046
33	43.745	47.400	50.725	54.776	57.648	60.395
34	44.903	48.602	51.966	56.061	58.964	61.738
35	46.059	49.802	53.203	57.342	60.275	63.076
36	47.212	50.998	54.437	58.619	61.581	64.410
37	48.363	52.192	55.668	59.892	62.883	65.739
38	49.513	53.384	56.896	61.162	64.181	67.063
39	50.660	54.572	58.120	62.428	65.476	68.383
40	51.805	55.758	59.342	63.691	66.766	69.699
45	57.505	61.656	65.410	69.957	73.166	76.232
50	63.167	67.505	71.420	76.154	79.490	82.664
60	74.399	79.087	83.305	88.386	91.957	95.857
70	85.529	90.537	95.031	100.432	104.227	107.812
80	96.581	101.885	106.636	112.336	116.329	120.107
90	107.568	113.151	118.144	124.125	128.307	132.262
100	118.501	124.348	129.570	136.815	140.178	144.300
110	129.388	135.487	140.925	147.423	151.958	156.238
120	146.571	152.222	157.389	163.678	168.122	172.451
140	168.618	174.659	180.174	186.875	191.604	196.099
160	190.522	196.926	202.766	209.852	214.845	219.588
180	212.310	219.056	225.200	232.647	237.890	242.866

TABLE 4 (a)

F distribution ($\alpha = 0.05$)

The entries in this table are critical values of F for which the area under the curve to the right is equal to 0.05.



Degrees of freedom for denominator	Degrees of freedom for numerator: 1	2	3	4	5	6	7	8	9	10
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	7.78	2.62	2.51	2.42	2.36	2.30	2.75
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

TABLE 4 (a) continued

F distribution ($\alpha = 0.05$)

Degrees of freedom for denominator	Degrees of freedom for numerator: 12	15	20	24	30	40	60	120	∞
1	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

APPENDIX 2: LIST OF KEY FORMULAE

MEASURES OF CENTRAL LOCATION

Arithmetic mean

Ungrouped data

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad 3.1$$

Grouped data

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{n}$$

Mode

Grouped data

$$M_o = O_{mo} + \frac{c(f_m - f_{m-1})}{2f_m - f_{m-1} - f_{m+1}} \qquad 3.3$$

Median

Grouped data

$$M_e = O_{me} + \frac{c\left[\frac{n}{2} - f(<)\right]}{f_{me}} \qquad 3.2$$

Lower quartile

Grouped data

$$Q_1 = O_{q1} + \frac{c\left(\frac{n}{4} - f(<)\right)}{f_{q1}} \qquad 3.7$$

Upper quartile

Grouped data

$$Q_3 = O_{q3} + \frac{c\left(\frac{3n}{4} - f(<)\right)}{f_{q3}} \qquad 3.8$$

Geometric mean

Ungrouped data

$$GM = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n} \qquad 3.4$$

Weighted arithmetic mean

Grouped data

$$\text{weighted } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} \qquad 3.5$$

MEASURES OF DISPERSION AND SKEWNESS

Range	Range = Maximum value – Minimum value + 1 $= x_{max} - x_{min} + 1$	3.9
Variance	*Mathematical – ungrouped data* $s^2 = \frac{\Sigma(x_i - \bar{x})^2}{(n-1)}$	3.10
	Computational – ungrouped data $s^2 = \frac{\Sigma x_i^2 - n\bar{x}^2}{(n-1)}$	3.11
Standard deviation	$s = \sqrt{s^2}$	3.12
Coefficient of variation	$CV = \frac{s}{\bar{x}} \times 100\%$	3.13
Pearson's coefficient of skewness	$sk_p = \frac{n\Sigma(x_i - \bar{x})^3}{(n-1)(n-2)s^3}$	3.14
	$sk_p = \frac{3\,(\text{Mean} - \text{Median})}{\text{Standard deviation}}$ (approximation)	3.15

PROBABILITY CONCEPTS

Conditional probability	$P(A/B) = \frac{P(A \cap B)}{P(B)}$	4.2
Addition rule	*Non-mutually exclusive events* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	4.3
	Mutually exclusive events $P(A \cup B) = P(A) + P(B)$	4.4

Multiplication rule	*Statistically dependent events* $P(A \cap B) = P(A/B) \times P(B)$	4.5
	Statistically independent events $P(A \cap B) = P(A) \times P(B)$	4.6
$n! = n$ factorial	$n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$	4.8
Permutations	${}_nP_r = \frac{n!}{(n-r)!}$	4.10
Combinations	${}_nC_r = \frac{n!}{r!\,(n-r)!}$	4.11

PROBABILITY DISTRIBUTIONS

Binomial distribution	$P(x) = {}_nC_x\, p^x(1-p)^{(n-x)}$ for $x = 0, 1, 2, 3, \ldots, n$	5.1
	$P(x\ successes) = \frac{n!}{x!\,(n-x)!} p^x(1-p)^{(n-x)}$ for $x = 0, 1, 2, 3, \ldots, n$	
Binomial descriptive measures	Mean $\mu = np$ Standard deviation $\sigma = \sqrt{np(1-p)}$	5.2
Poisson distribution	$P(x) = \frac{e^{-a} a^x}{x!}$ for $x = 0, 1, 2, 3 \ldots$	5.3
Poisson descriptive measures	Mean $\mu = a$ Standard deviation $\sigma = \sqrt{a}$	5.4
Standard normal probability	$z = \frac{x - \mu}{\sigma}$	5.6

CONFIDENCE INTERVALS

Single mean *n large; variance known*

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z\frac{\sigma}{\sqrt{n}} \qquad 7.1$$

(lower limit) (upper limit)

n small; variance unknown

$$\bar{x} - t_{(n-1)}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{(n-1)}\frac{s}{\sqrt{n}} \qquad 7.2$$

(lower limit) (upper limit)

Single proportion

$$p - z\sqrt{\frac{p(1-p)}{n}} \leq \pi \leq p + z\sqrt{\frac{p(1-p)}{n}} \qquad 7.3$$

(lower limit) (upper limit)

HYPOTHESES TESTS

Single mean *Variance known*

$$z\text{-}stat = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \qquad 8.1$$

Variance unknown; n small

$$t\text{-}stat = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \qquad 8.2$$

Single proportion

$$t\text{-}stat = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \qquad 8.3$$

Difference between two means *Variances known*

$$z\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad 9.1$$

Variances unknown; n_1 and n_2 small

$$t\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \qquad 9.2$$

Paired t-test

$$t\text{-}stat = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}} \qquad 9.5$$

where $\mu_d = (\mu_1 - \mu_2)$

and $s_d = \sqrt{\frac{\Sigma(x_d - \bar{x}_d)^2}{n-1}}$

Differences between two proportions

$$z\text{-}stat = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\hat{\pi}(1-\hat{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } \hat{\pi} = \frac{x_1 + x_2}{n_1 + n_2};\ p_1 = \frac{x_1}{n_1};\ p_2 = \frac{x_2}{n_2} \qquad 9.8$$

Chi-Squared

$$\chi^2\text{-}stat = \Sigma\frac{(f_o - f_e)^2}{f_e} \qquad 10.1$$

Overall mean

$$\bar{\bar{x}} = \frac{\Sigma\Sigma x_{ij}}{N} \qquad 11.2$$

Total sum of squares (SSTotal)

$$= \sum_i \sum_j (x_{ij} - \bar{\bar{x}})^2 \qquad 11.3$$

SST

$$= \sum_j^k n_j(\bar{x}_j - \bar{\bar{x}})^2 \qquad 11.4$$

SSE

$$= \sum_i \sum_j (x_{ij} - \bar{x}_j)^2 \qquad 11.5$$

SSTotal $= \text{SST} + \text{SSE}$ 11.6

MSTotal $= \frac{\text{SSTotal}}{N-1}$ 11.7

MST $= \frac{\text{SST}}{k-1}$ 11.8

MSE $= \frac{\text{SSE}}{N-k}$ 11.9

F-stat $= \frac{\text{MST}}{\text{MSE}}$ 11.10