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UNIVERSITY OF SWAZILAND
FACULTY OF EDUCATION
FINAL EXAMINATION PAPER 2005

TITLE OF PAPER:	CURRICULUM STUDIES IN MATHEMATICS
COURSE CODE:	EDC 281
STUDENTS:	B.ED II AND PGCE
TIME ALLOWED:	THREE (3) HOURS
INSTRUCTIONS:	ATTEMPT **ALL FIVE** QUESTIONS EACH QUESTION IS WORTH 20 MKS
ADDITIONAL MATERIALS:	APPENDIX 1 - 5 (30 PAGES)

THIS PAPER CONTAINS **TWO** PAGES. DO NOT OPEN UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

EDC 281 Final 2005

ANSWER **ALL** QUESTIONS

Question 1.

Prepare a scheme of work for the **two** form three chapters in appendix 1. [20]

Question 2.

Write a one hour **practical** lesson plan for the sub-topic 'surface area of prisms' from the scheme of work in question 1. Use the **'guided discovery'** approach to learning for this lesson. [20]

Question 3.

(a) For the topic 'Probability' write **five** cognitive objectives on each of the following:-
(i) Comprehension (ii) Application. [10]
(b) For the topic 'Quadratic Equations' construct a question for each of the following:-
(i) Comprehension (ii) Application. [10]

Question 4.

Read the story in appendix 4 and do the following:-

(i) Draw a graph to represent Nozipho's journey to and from school. [5]
(ii) Using the graph and the story write at least **five** questions that you could use in this topic. [5]
(iii) Write **one** project assignment which makes use of pupils' everyday experiences, to introduce an IGCSE topic of your choice. You should explain why you consider this project as relevant to pupils' lives. [5]
(iv) List **five** objectives of learning this topic that the above project would cover. [5]

Question 5.

(a) Explain the following words/phrases:-
(i) Ausubel's meaningful learning. [3]
(ii) A concept map. [3]
(iii) Subsumers. [2]
(iv) Advance organisers. [2]
(b) (i) Do the investigation in appendix 5. [7]
(ii) Give the topics of Mathematics that are covered by this investigation. [3]

PRISMS

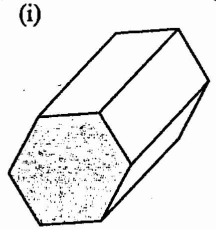
4

4.1 A reminder

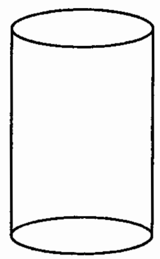
In Book 2 we met prisms. Prisms are solids with uniform cross-sectional area.

1 **(a)** Name the following prisms.

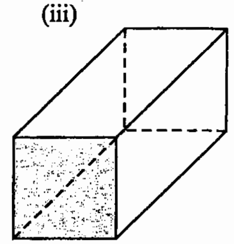
(i)



(ii)



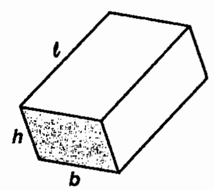
(iii)



(b) Sketch the nets of prisms in **(a)**.

4.2 Volume of a prism

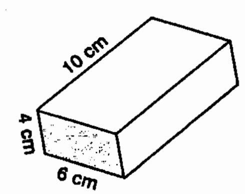
The volume of the cuboid in the diagram below is lbh. The area of its cross section is bh.



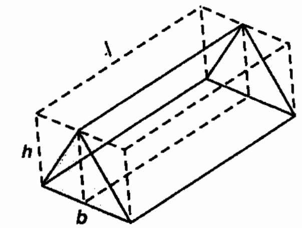
> The volume of a cuboid is given by *area of cross section* × *length.*

2 The cuboid in the diagram below has length 10 cm, breadth 6 cm and height 4 cm.

(a) Calculate the area of the shaded end.
(b) Calculate the volume of the cuboid.



3 The diagram shows a triangular prism enclosed in a cuboid of length l, breadth b and height h.



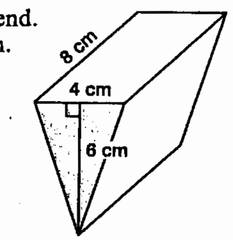
(a) What fraction, of the area of the end of the cuboid, is the area of the shaded triangle?
(b) What fraction, of the volume of the cuboid is the volume of the triangular prisms?
(c) Write a formula for the volume of the triangular prism.

Did you find that the formula for the volume of a triangular prism is $\frac{1}{2}lbh$?
We can write this as $\frac{1}{2}bh \times l$ or $\frac{1}{2} \times b \times h \times l$ or $\frac{b \times h \times l}{2}$

> The volume of a triangular prism is *area of the cross section* or *end* × *length.*

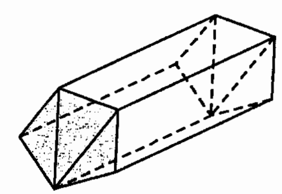
4 In the triangular prism given:

(a) Find the area of the shaded end.
(b) Find the volume of the prism.



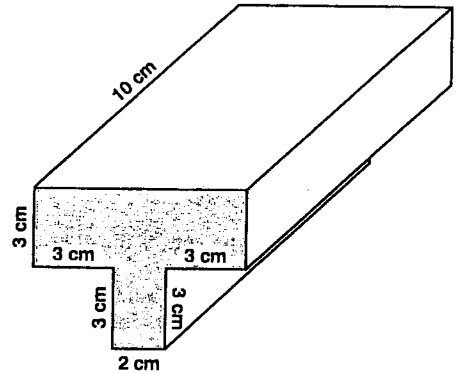
5 Here is a pentagonal prism which has been divided into three triangular prisms.

(a) How can we find the volume of each triangular prism?
(b) How can we find the volume of the pentagonal prism?

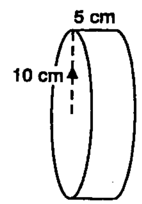


> The volume of any prism is the *area of the cross section* or *end* × *length.*

6 Calculate the volume of the prism below. All measurements are in centimetres. (Find the area of the shaded end first.)

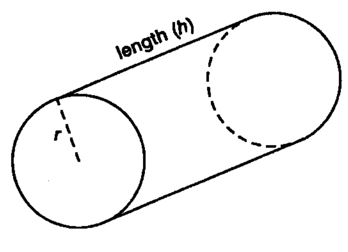


7 The diagram shows a cylinder (circular prism) of radius 10 cm and height 5 cm.



(a) Calculate the area of one end, taking π to be 3.14.
(b) Calculate the volume of the cylinder.

8 The cylinder below has a radius of r and length h units.



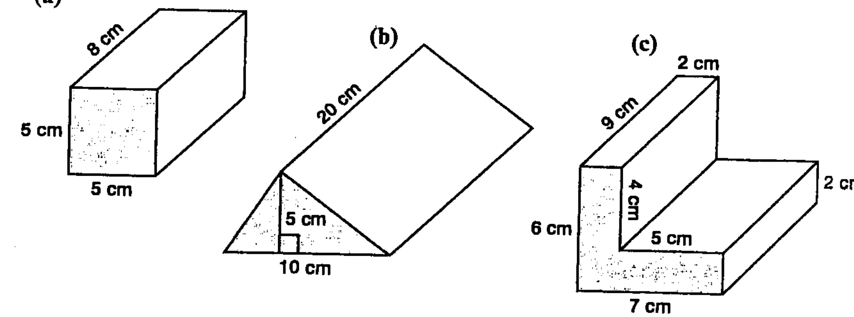
(a) What is the area of its end?
(b) Write the formula for the volume, V, of the cylinder.

The formula for the volume of a cylinder is $V = \pi r^2 h$.

The volume of a prism is given by *cross-sectional area* × *length*.

Exercise 4.2

1 Calculate the volumes of the following prisms.



2 The end of a prism 11 cm long has an area of 40 cm^2. Calculate its volume.

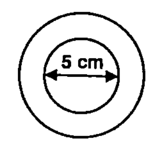
3 A prism of volume 160 cm^3 has a cross-sectional area of 16 cm^2. Calculate its length.

4 A prism of volume 240 cm^3 has length of 8 cm. Calculate its cross-sectional area.

5 Taking π as 3.14, calculate the volume of the following cylinders:
(a) radius 10 cm, length 50 cm
(b) radius 5 cm, length 20 cm
(c) diameter 6 cm, length 10 cm
(d) diameter 5 cm, length 40 cm

6 A condensed milk tin has an inside diameter of 7 cm and height 10 cm. Calculate the greatest volume of milk which the tin can hold. Take π as $\frac{22}{7}$.

7 The internal diameter of a metal pipe 6 m long is 5 cm. Calculate the greatest volume of water the pipe can hold. Take π as 3.14.



8 A coin 10 mm in diameter is 2 mm thick. Calculate the volume of metal in the coin. Take π as 3.14.

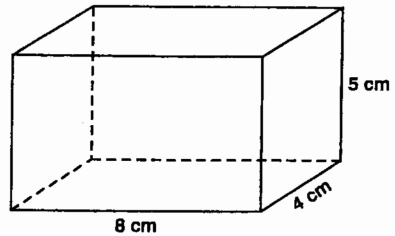
9 A cylindrical bucket 28 cm in diameter has a height of 40 cm. The bucket is filled with water. Calculate the volume of water in the bucket. Take π as $\frac{22}{7}$.

10 Take π as 3.14 in this question. A cylindrical can of diameter 12 cm contains water to the depth of 21 cm.
(a) What volume of water to the nearest cm^3 is in this can?
(b) The water is poured into a rectangular container whose base is 16 cm by 9 cm. Calculate the depth of water in the container correct to three significant figures.

4.3 Surface area of a prism

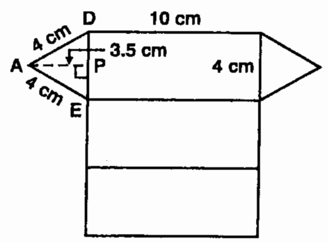
Earlier in this series we met nets of prisms; in this section we will deal with the areas of nets of prism. The total surface area of a prism consists of the area of all the faces of the prism (i.e. the area of the net of the given prism).

9 The diagram shows a closed box 8 cm by 5 cm by 4 cm.



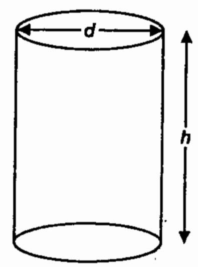
(a) Sketch the net of the box.
(b Calculate the total surface area of the box.

10 The diagram shows the net of a triangular prism with each end an equilateral triangle of side 4 cm. The length of the prism is 10 cm. The perpendicular distance $AP = 3.5$ cm. (This distance has been rounded to one decimal place.)



(a) Calculate the area of triangle ADE.
(b) Calculate the surface area of the prism.

11 The diagram below shows a cylinder closed at both ends.



(a) Sketch the net of the prism.
(b) How many shapes does the net consist of?
(c) Find the area of each shape.
(d) Find the total surface area of the prism.

You should realise that the closed cylinder consists of a rectangle (bent to form the curved surface) and two circles. The length of the cylinder is a circumference of the circle.

So the total surface area of a closed cylinder $= \textit{curved area} + \textit{area of two circles}$
$= \pi dh + 2\pi r^2$

12 A closed cylinder has a diameter of 14 cm and a height of 8 cm. Taking π as $\frac{22}{7}$ calculate the following:

(a) the curved surface area of the cylinder
(b) the area of one end of the cylinder
(c) the total surface area of the cylinder

Exercise 4.3

1 Take π as $\frac{22}{7}$ in this question. Calculate the curved surface areas of cylinders having:

(a) radius 21 cm, height 5 cm
(b) diameter 8 cm, length 14 cm

2 Take π as 3.14 in this question. Calculate the curved surface areas of cylinders having:

(a) radius 5 cm, height 8 cm
(b) diameter 20 cm, length 100 cm

3 Taking π as 3.14, calculate the total surface area of a cylinder of radius 2 cm and length 50 cm.

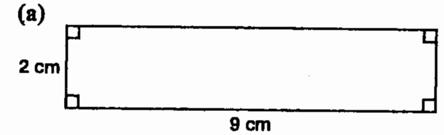
4 A cylindrical milk tin is open at one end. The diameter of the tin is 10 cm and its height is 15 cm. Calculate the outside surface area of the open tin taking π as 3.14.

5 The curved surface of a cylindrical pipe is to be painted. What area is to be painted if the pipe has the radius of 10 cm and is 10 m long? Take π as 3.14. Give your answer in:

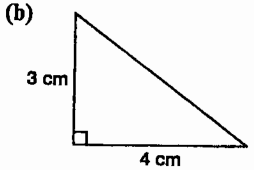
(a) square centimetres
(b) square metres

6 Each of the following diagrams shows one end of a prism which is 30 cm long.

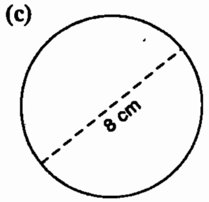
(a)



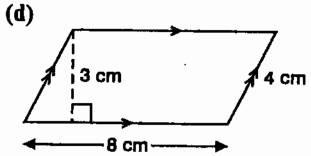
(b)



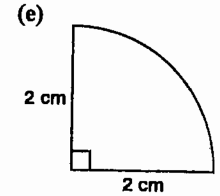
(c)



(d)



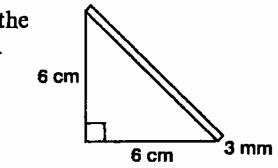
(e)



For each prism calculate, taking π as 3.14 where necessary:

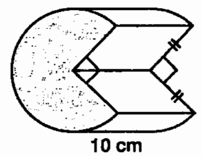
(a) the total surface area
(b) the volume

7 In a metal set square, 3 mm thick, the arms of the right angle are each 6 cm long. The metal has a density of 6.5 g/cm^3. Calculate the mass of the set square. *Hint*: Density $= \frac{\text{mass}}{\text{volume}}$



8 A metal rod 4 cm in diameter and 10 cm long has a groove cut from it as shown. Taking π as 3.14, calculate the:

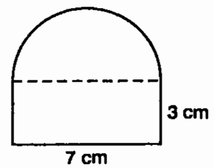
(a) area of the end
(b) volume of the metal remaining



9 Closed food cans have a radius of 7 cm and a height of 10 cm.

(a) Taking π as $\frac{22}{7}$, calculate the area, in square metres, of metal sheet in 10 000 cans.
(b) When buying the metal sheet, five more is allowed for wastage. What area of metal sheet must be bought to make the 10 000 cans?

10 The diagram below shows the cross section of a prism 5 cm long. It is made up of a semi-circle on the long side of a 7 cm by 3 cm rectangle. A factory makes these solid prisms from metal alloy of density 3.8 g/cm^3. How many prisms can be made from 10 kg of metal? Take π as $\frac{22}{7}$.



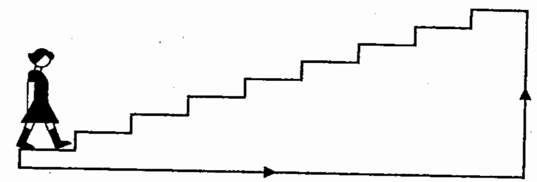
Chapter review

- A prism is a solid with uniform cross-sectional area.
- The volume of a prism is given by *cross-sectional area* × *length* (perpendicular to the base).
- The volume of a cylinder, $V = \textit{base area} \times \textit{height}$
  $V = \pi r^2 h$
- The total surface area of a prism is given by the sum of the areas of its faces.
- The net of a closed cylinder consists of a rectangle and two congruent circles.
- A closed cylinder consists of a curved surfaces and two circles on the ends, hence the total surface area of a closed cylinder $= 2\pi r^2 + 2\pi rh$
  $= 2\pi r(r + h)$

GRADIENTS, GRAPHS AND EQUATIONS OF STRAIGHT LINES

5

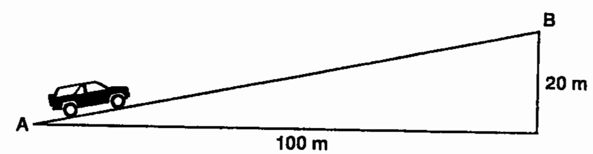
5.1 Gradients



When we walk up a staircase we move in two directions; one direction is horizontal and the other is vertical. The person in the picture above moves upwards (vertical) and also to the right (horizontal). A movement that is both vertical and horizontal can be represented by a sloping line.

■ ***Example***

The car in the figure below moves from point A to point B. What are the horizontal and vertical distances that the car would have covered when it reaches point B?



From point A to point B the car would have moved 20 metres upwards (vertical) and 100 metres to the right (horizontal).

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1 In your Workbook, draw the lines represented by the following equations:

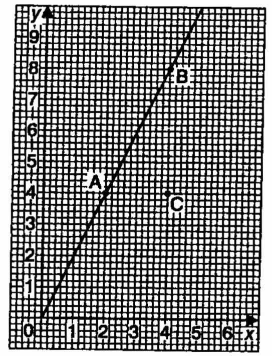
(a) $y = x$ **(b)** $y = 2x$ **(c)** $y = 3x$ **(d)** $y = \frac{1}{2}x$

Which is the steepest line?
Which line is least steep?
Write the equations in order of the steepness of the lines.

2 On the same diagram, draw the graphs of $y = -x$ and $y = -2x$.
The lines you drew in **1** go upward as they go to the right.
What happens to the lines in **2**?

5.2 Calculating the gradient

Your graph of $y = 2x$ should look like this. (We are ignoring negative values for x and y in this graph.)



Take the two marked positions A and B on the line. Notice that for every two points of the run (horizontal distance), there is a rise (vertical distance) of four points. The ratio of rise to run is called the *gradient*. The gradient of a line is the *measure of its steepness* or *rate of change*.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$$

The gradient is sometimes called the *slope of the line*.

To calculate the gradient of $y = 2x$, first take two points A and B on the line. The coordinates of A are (2, 4) and of B are (4, 8).

In the graph of $y = 2x$ the vertical change is the change in y-axis and the horizontal change is a change in the x-axis.

$$\text{So gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Where (x_1, y_1) refers to the first set of coordinates and (x_2, y_2) refers to the second set. In our case (x_1, y_1) is for A(2, 4) and (x_2, y_2) represents (4, 8).

Gradient $= \frac{BC}{AC} = \frac{8-4}{4-2} = \frac{4}{2} = 2$

> The subtractions must be done in the same order as in the case above; second point coordinates minus first point coordinates.

3 Choose any two other points on the line $y = 2x$ and calculate the gradient. You should get the same answer as above.

4 On the line $y = 3x$, mark the points C(1, 3) and D(3, 9).
Use the method above to find the gradient of this line.
Take any two other points and calculate the gradient.

5 Find the gradients of the lines $y = x$ and $y = \frac{1}{2}x$ by similar methods.

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6 On your Workbook diagram that you used for question **1**, draw the line given by the equation $y = 2x + 1$.
How is it related to the line $y = 2x$?
Check that the points E(1, 3) and F(2, 5) are on this line.
The gradient is given by $\frac{5-3}{2-1} = \frac{2}{1} = 2$.

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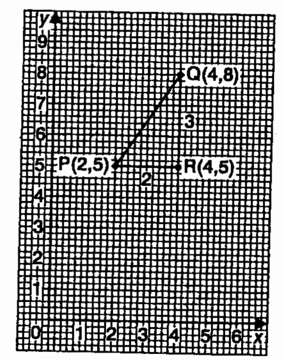
7 On the Workbook diagram that you used for question **1**:
Draw a line parallel to $y = 3x$ and find its gradient.
You should find that parallel lines have equal gradients.
The gradient of a **non-vertical** line can be found if two points on the line are known.

■ ***Example***

Calculate the gradient of the line joining P(2, 5) and Q(4, 8).

QR = 8 − 5 = 3 and PR = 4 − 2 = 4

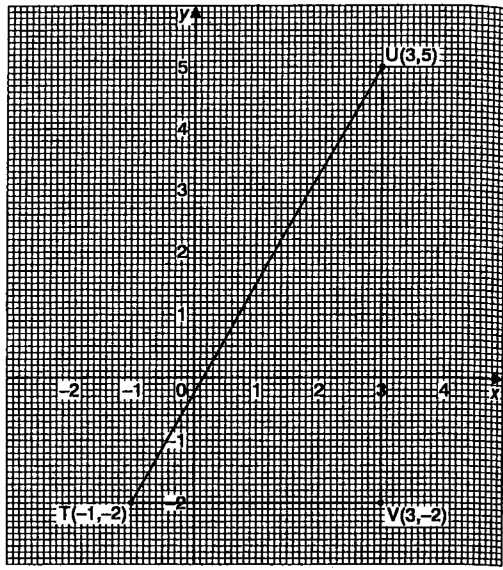
$$\text{Gradient} = \frac{QR}{PR} = \frac{8-5}{4-2} = \frac{3}{2} = 1\tfrac{1}{2}$$



■ ***Example***

Repeat the previous example but with T(−1, −2), and U(3, 5). VU = 5 − (−2) = 7 and TU = 3 − (−1) = 4

$$\text{Gradient} = \frac{UV}{TU} = \frac{5-(-2)}{3-(-1)} = \frac{7}{4} = 1\tfrac{3}{4}$$



■ ***Example***

Find the gradient of the line joining V(2, 3) and W(4, 3).

Gradient $= \frac{3-3}{4-2} = \frac{0}{2} = 0$

> Lines parallel to the x-axis have a *zero* gradient.

■ ***Example***

Find the gradient of the line joining the points P(2, 3) and Q(2, 4).

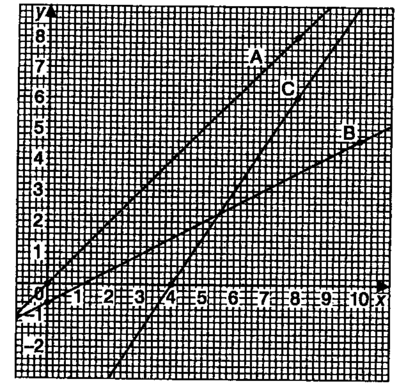
Gradient $= \frac{4-3}{2-2} = \frac{1}{0}$, but we know that division by zero is excluded.

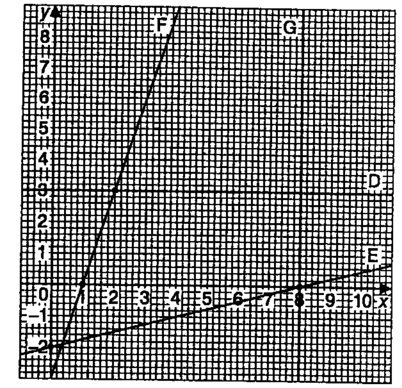
So the gradient is undefined.

> The gradient of a line parallel to the y-axis is undefined.

Exercise 5.2

1 Find the gradients of the lines shown in the figure below.

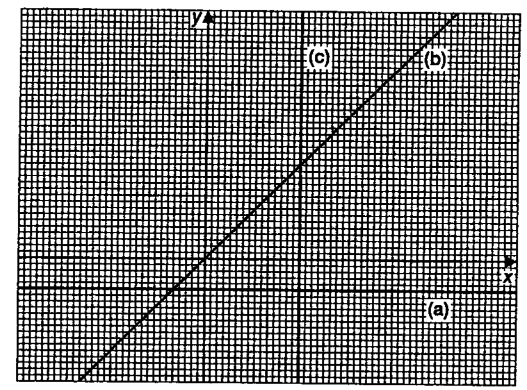




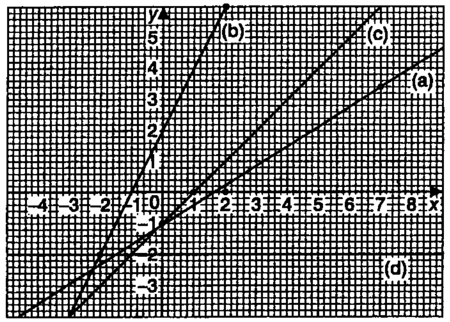
What is the gradient of:

(a) the x-axis? **(b)** the y-axis?

2 For each line in the figure below, state whether the gradient is positive, negative, zero or undefined.



3 The gradients of the lines lettered below are 0, 1, $\frac{2}{3}$ and 2, respectively. Match each line with its gradient.



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4 Draw the following straight line graphs on the grid in your Workbook.

(a) gradient 2, passing through (0, 0)
(b) gradient 1, passing through (0, 1)
(c) gradient $\frac{1}{2}$, passing through (2, 0)
(d) gradient 0, passing through (2, 4)
(e) gradient 3, passing through (1, −3)

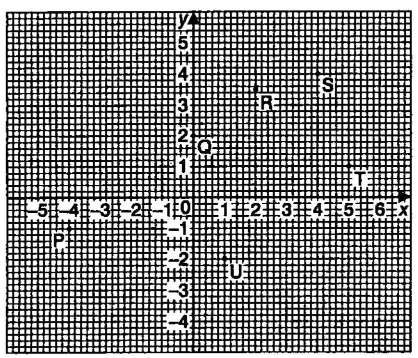
5 Find the gradients of the lines joining the following pairs of points.

(a) (0, 0) and (2, 6)
(b) (1, 2) and (3, 4)
(c) (−1, −5) and (0, 12)
(d) (−1, 2) and (3, 3)
(e) (0, −2) and (5, −2)
(f) (−2, −6) and (3, 4)
(g) (−1, −2) and (−1, −3)
(h) (4, −3) and (4, −1)

6 Calculate the gradients of the lines joining the following pairs of points.

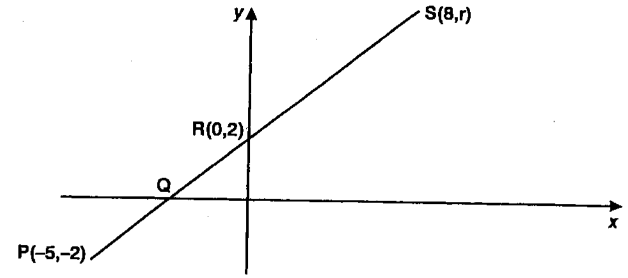
(a) P and Q (b) P and R (c) Q and S (d) T and U

What conclusions can you make from your results?



7 PQRS is a straight line.

(a) Find the coordinates of Q.
(b) Calculate the value of r, the y-coordinate of point S.



5.3 Negative gradients

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At the beginning of this chapter (2), you drew the graphs of $y = -x$ and $y = -2x$. Find these graphs in your Workbook.

8 Mark the points A(1, −1) and B(3, −3) on $y = -x$. Calculate the gradient. You should get $\frac{-2}{2} = -1$.

9 Find the gradient of the line $y = -2x$ by a similar method.

■ ***Example***

Find the gradient of the line joining A(1, 6) and B(3, 2).

Gradient $= \frac{2-6}{3-1} = \frac{-4}{2} = -2$

The subtractions must be done in the same order; in this case, **second point coordinates minus first point coordinates.**

A useful check on the sign of the gradient is to draw a rough sketch.

> If the line slopes downwards as it goes to the right it has a *negative* gradient.

Exercise 5.3

1 Draw the following lines on the same axes. Choose two points and calculate the gradient for each.

(a) $y = 4 - 2x$
(b) $y = 6 - x$
(c) $y = 8 - 1\frac{1}{2}x$
(d) $3y = 6 - 2x$
(e) $y = 6 - 2x$
(f) $y = 8 - x$

2 Calculate the gradient of the lines joining the following pairs of points.

(a) (2, 1) and (3, 0)
(b) (−1, 2) and (2, 0)
(c) (−2, −3) and (0, −4)
(d) (−3, 8) and (2, −2)
(e) (−3, 0) and (0, −3)
(f) (2, 4) and (−1, 13)

5.4 The gradient-intercept form of the equation of a straight line

The coefficient of x is the number multiplying x in an equation or expression.

■ ***Example***

In the equation $y = 3x$, the coefficient of x is 3.
In the equation $y = 5 - 2x$, the coefficient of x is -2.
Compare the gradient and the coefficient of x in the equations in questions [3], [4], [5], [6] and [7] in section 5.2.

You should find that the gradient is the same as the coefficient of x in the above cases.

10 On a sheet of squared paper, plot the graph for the equation $y = 3x - 1$, for values of x from -1 to 3. Check that the points (1, 2) and (3, 8) are on the line.
Now, check by substituting the points (1, 2) and (3, 8) into the equation $y = 3x - 1$.
The gradient is $\frac{8-2}{3-1} = \frac{6}{2} = 3$.

> The gradient is the coefficient of x.

11 On a sheet of squared paper, plot the graph for the equation $y = 3 - 2x$, for the values of x from -1 to 3. Check that the points (1, 1) and (3, -3) are on the line.
Now, check by substituting the points (1, 1) and (3, -3) into the equation $y = 3 - 2x$.
The gradient is $\frac{-3-1}{3-1} = \frac{-4}{2} = -2$.

> Where the coefficient of y is 1, the gradient is equal to the coefficient of x, and is usually denoted by the letter m.

Calculating the y-intercept

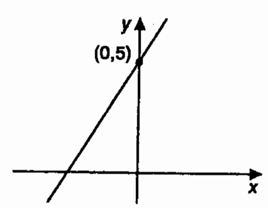
Any straight line which is not parallel to the y-axis will cut across the y-axis at some point, called the *y-intercept*.

> The y-intercept is the distance from the origin to the point where a line cuts the y-axis.

Consider the line with equation $y = 2x + 5$.
At the point where this line crosses the y-axis, $x = 0$.

When $x = 0$
$y = 2x + 5$
$y = 0 + 5$
$y = 5$

The line cuts the y-axis at the point (0, 5) and so the y-intercept is 5.

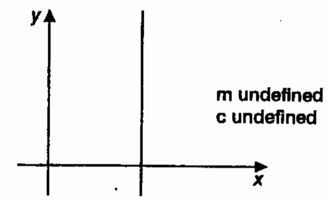


12 What is the y-intercept of each of the following lines:
(a) $y = 2x + 3$? (b) $y = 3x + 6$? (c) $y = 5x - 7$?
(d) $y = 6 - 2x$? (e) $2y = 4x + 3$?

In each of the lines where the coefficient of y is 1, the number or constant in the equation is the y-intercept.

> The equation $y = mx + c$ is known as the **gradient-intercept** form of the equation of a **non-vertical** straight line; m is the gradient, c is the y-intercept.

Note that the equation of a straight line must be in this form with y (coefficient 1) the subject, if the gradient and y-intercept are to be obtained by inspection.



The equation of a vertical line cannot be written in the form $y = mx + c$ as m is not defined, and the line (if it is not the y-axis) won't cut the y-axis, so c is undefined.

■ ***Example***

What is:
(i) the gradient?
(ii) the y-intercept of the line with equation $2y - 6x = 5$?

$2y - 6x = 5$
$2y = 6x + 5$
$y = 3x + \frac{5}{2}$ (gradient-intercept form)

By inspection: gradient = 3, y-intercept = $\frac{5}{2}$ or $2\frac{1}{2}$.

13 Write down the gradient and y-intercept for each of the following equations (rearrange where necessary).
(a) $y = 2x + 3$ (b) $y = x - 2$ (c) $y = -2x - 3$
(d) $y = 4 - 3x$ (e) $y = 2$ (f) $x + y = 1$
(g) $2y = 3x - 1$ (h) $3x + 4y = 0$ (i) $2x = 3y + 4$

14 Write down the equation of the line in each of the following cases:
(a) gradient = $\frac{-4}{3}$, y-intercept = 45
(b) gradient = $\frac{2}{5}$, y-intercept = 8.5
(c) gradient = $\frac{9}{4}$, y-intercept = -25

> Parallel lines have the same gradient.
> Parallel lines do not intersect.

Exercise 5.4

1 Find the equations of the following lines:

	Gradient	Passing through
(a)	1	(0, 4)
(b)	3	(0, 4)
(c)	−1	(0, 2)
(d)	0	(0, 8)

2 Find the equations of lines *parallel* to the given lines and passing through the given point.

(a) $y = 2x + 1$ (0, 3) **(b)** $y = 4 - x$ (0, 2)
(c) $y = \frac{3x}{4} + 2$ (0, −4) **(d)** $2y = x - 2$ (0, 4)
(e) $4y = 3x$ (0, −6)

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3 On the grid in your Workbook, draw, in each case, a line *parallel* to the given line and passing through the given point.

(a) $y = 3x - 5$ (1, 2) **(b)** $y = 4x + 3$ (2, 6)
(c) $y = x - 4$ (−3, 1) **(d)** $3y = 2x$ (1, −1)
(e) $2y = -3x - 1$ (2, 0)

4 Find the gradient of the lines:

(a) $y = 6$ **(b)** $y = -6$

5.5 More equations of lines

■ ***Example***

Find the equation of the line with gradient −2, passing through the point (1, 4).
$y = -2x + c$
Substitute (1, 4) in the equation:
$4 = -2(1) + c$ so $c = 6$
The equation is therefore $y = -2x + 6$ or
$y = 6 - 2x$

■ ***Example***

Find the equation of the line joining (2, 1) and (3, 4).
The gradient of the line is $\frac{4-1}{3-2} = \frac{3}{1} = 3$. So $m = 3$.
The equation is $y = 3x + c$.

Now substitute (2, 1) into the equation.
$1 = 3 \times 2 + c$
so $c = -5$
The equation is $y = 3x - 5$.
This can be checked using the other point (3, 4).
$3x - 5 = (3 \times 3) - 5 = 4$ which is the y-value and so our result is correct.

Exercise 5.5

1 Find the equations of the following lines and write down the y-intercept (where it cuts the y-axis).

	Gradient	Passing through
(a)	2	(1, 3)
(b)	3	(−1, 4)
(c)	$1\frac{1}{2}$	(3, −2)
(d)	1	(1, 0)

2 Find the equations of the lines passing through the following pairs of points:

(a) (0, 2) and (3, 8) **(b)** (0, 2) and (2, 0)
(c) (−1, 3) and (5, 0) **(d)** (3, 4) and (4, 7)
(e) (2, 5) and (−2, −1) **(f)** (1, 4) and (2, 1)
(g) (2, −1) and (−1, 1) **(h)** (5, 4) and (6, 4)

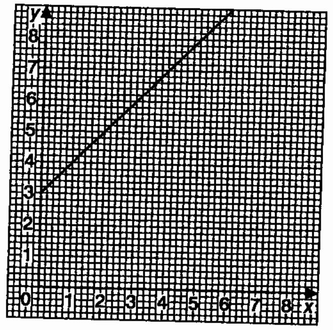
3 **(a)** Draw the line $x = 2$ (parallel to the y-axis). What is its gradient and y-intercept?
(b) Draw the line $y = 4$ (parallel to the x-axis). What is its gradient and y-intercept?

5.6 Finding equations of lines from graphs

A line might be given as a graph. In this case choose two convenient points on the line and use the methods above to find the gradient m and the y-intercept c. Then the equation of the line is $y = mx + c$. **This applies only to non-vertical lines.**

■ ***Example***

Find the equation of the line.

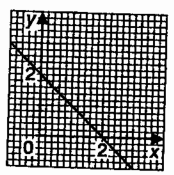


Two convenient points are (0, 3) and (5, 8).
The gradient is $\frac{8-3}{5-0} = \frac{5}{5} = 1$.
The equation is $y = x + c$. As the y-intercept is 3, $c = 3$.
The equation is $y = x + 3$.
Check that (1, 4) is on the line. Does it satisfy the equation?
If $x = 1$, $y = 1 + 3$, $y = 4$. So the equation is satisfied. (LHS = RHS)

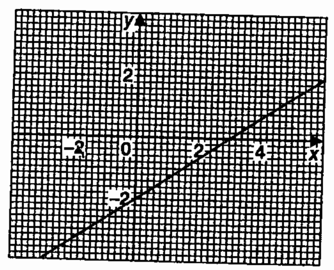
Exercise 5.6

1 For each diagram given below, find the equation of the line.

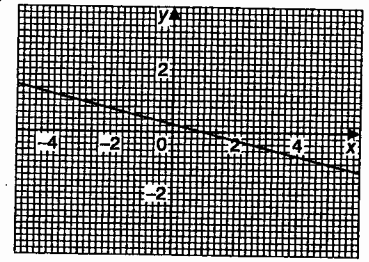
(a)



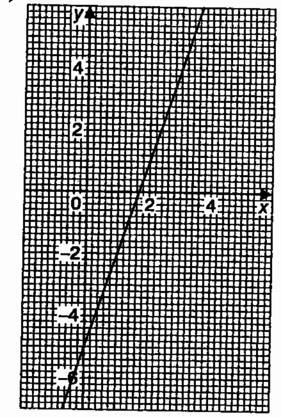
(b)



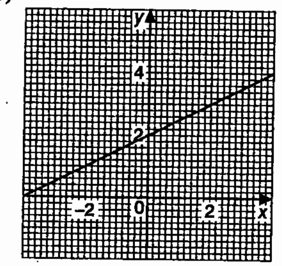
(c)



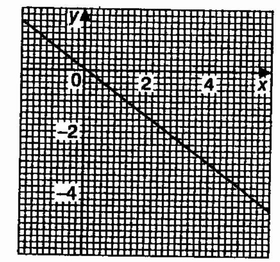
(d)



(e)



(f)



Choose a third point on each line and check that it fits your equation.

Exercise 5.6M (Miscellaneous)

1 Write down the equation of the line with gradient 4 which passes through the origin.

2 A straight line passes through the points (0, 4) and (1, 7). Find:
(a) the gradient of the line
(b) the x-intercept of the line

3 Given that the point (2, 4) lies on the line with equation $ax + y = 8$, find a. Rewrite the equation in the form $y = mx + c$, and state the gradient.

4 Write down the equation of the straight line parallel to $y = 1 - 2x$, passing through the point (4, −2).

5 The line with equation $3x + 2y = 6$ meets the y-axis at P. Write down the coordinates of P.

6 The gradient of the line joining (p, 5) and (−2, 8) is $-\frac{1}{2}$. Calculate the value of p.

7 Given A(−2, 0), B(2, 0) C(0, 4) and D(−4, 4), show that AB is parallel to CD and AD is parallel to BC.

8 Find gradients of the lines:

(a) $\frac{x}{6} - \frac{y}{4} = 1$ (b) $6 + 2x - 4y = 0$

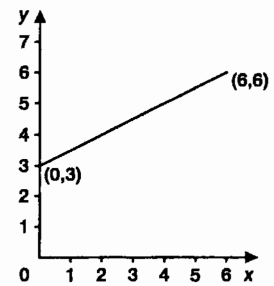
9 A straight line passes through the point (3, 5) and has a gradient of −2. Write down the coordinates of any other point of the line. CI, 76.

10 The straight line with equation $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points (2, 0) and (0, −5). Find a and b and the gradient of the line. CI, 76.

11 The equation of a straight line is $5x - y = 24$.

(a) Write down the gradient of the line.
(b) A point M lies on the line and is equidistant from the x-axis and the y-axis. Find the coordinates of M. CI, 77.

12 A straight line passes through the points (0, 3) and (6, 6). CI, 77.



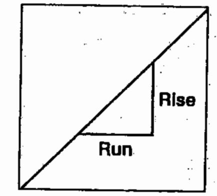
Find:

(a) the gradient of the line
(b) the equation of the line.

13 (a) The line $\frac{x}{6} + \frac{y}{8} = 1$ cuts the x-axis at H and the y-axis at K. CII, 78.
Find:
(i) the coordinates of H
(ii) the coordinates of K
(b) P is the point (8, 0) and Q is the point (0, 15). Calculate the length of PQ.

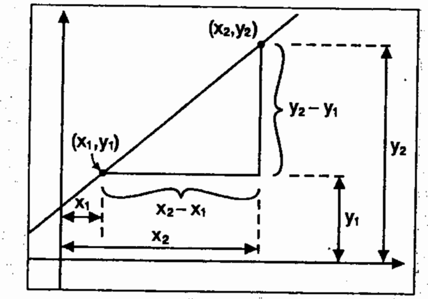
Chapter review

- The ratio of rise to run is called the gradient.

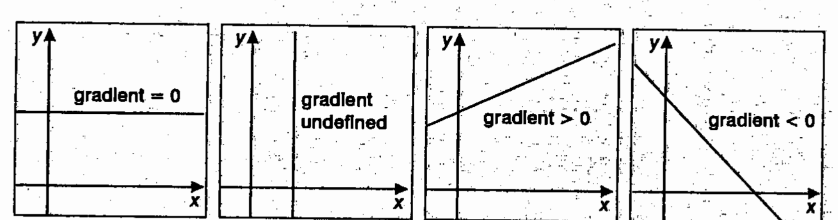


- The gradient is a measure of the steepness of a line.
- Gradient $= \frac{\text{change in } y}{\text{change in } x}$

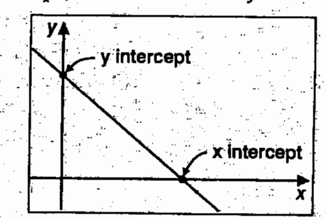
Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$



- The gradient of the x-axis is zero (and so is the gradient of any line parallel to the x-axis).
- The gradient of the y-axis is *not* defined (nor is the gradient of any line parallel to the y-axis).
- If a line slopes upward as we move to the right along the x-axis, it has a positive gradient.
- If a line slopes downward as we move to the right along the x-axis, it has a negative gradient.



- The coefficient of x is the number multiplying x in an equation.
- $y = mx + c$ is the gradient-intercept form of the equation of a (non-vertical) straight line, where m is the gradient, c is the y-intercept, the coefficient of y is 1 and y is the subject of the formula.
- The x-intercept is the point where the line cuts the x-axis.
- The y-intercept is the point where the line cuts the y-axis.



5.6d Combining independent events

Two events are described as **independent** if they have no effect on each other. Rolling dice, tossing coins and any other actions you can repeat in exactly the same way over and over again will give independent events.

If you choose two items **with replacement** then events will be independent because the second choice will not be affected by the first choice.

When you combine two trials and events are independent then all the different combinations of outcomes of the two events are possible. (This is not true when events are not independent. Think of being dealt two cards: they can't both be the Ace of Hearts.)

E.g. A drinks machine contains Coke, Pepsi, Tango, 7UP, Vimto and Citrus Spring. List all the possible combinations of drinks that Amit and Tim might choose, assuming that they choose independently of each other. (This is like choosing with replacement.)

You could list all the possible combinations in any order as you thought of them.

Coke and Coke
7UP and Vimto
Pepsi and Tango ...

But then it's very hard to see if you've missed a possible combination. It is better to list the combinations in a systematic way, following a pattern.
List all the combinations which have Coke first:

Coke and Coke
Coke and Pepsi
Coke and Tango
Coke and 7UP
Coke and Vimto
Coke and Citrus Spring

Then look at the next drink on the list and write down all the combinations which have this first.

Pepsi and Coke
Pepsi and Pepsi
Pepsi and Tango
Pepsi and 7UP
Pepsi and Vimto
Pepsi and Citrus Spring

Always use the same order for the second choice.

To calculate the number of possible combinations, you multiply the number of possible outcomes for the individual trials.

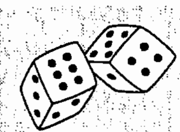
There are 6 ways for Amit to choose and 6 ways that Tim could choose. Tim's choice and Amit's choice are independent so whatever Amit chooses there are still 6 possible choices for Tim. The total number of ways Amit and Tim can choose their drinks is $6 \times 6 = 36$.

It is useful to work this out to check that you have found all the possible combinations. Another way to display all the possible outcomes is to use a table called a **sample space**.

Tim \ Amit	C	P	T	7	V	S
C	CC	CP	CT	C7	CV	CS
P	PC	PP	PT	P7	PV	PS
T	TC	TP	TT	T7	TV	TS
7	7C	7P	7T	77	7V	7S
V	VC	VP	VT	V7	VV	VS
S	SC	SP	ST	S7	SV	SS

The more possible outcomes each trial has the easier it is to find all the combinations using a sample space. You can clearly see all 36 combinations.

1 (a) How many different possible outcomes are there when you roll two dice?

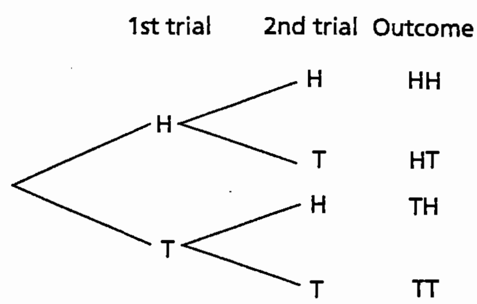


(b) Draw a sample space to show all the possible outcomes.

You can also display the possible outcomes of two events using a **tree diagram**.

E.g. This is a tree diagram showing the results of tossing a coin twice.

Remember: Put the outcomes of the second trial in the same order as the first.



2 (a) There are three cards A, B and C face down on a table. One card is picked up and then replaced and then a second card is picked up. List the possible ways in which they could be chosen.

(b) Complete this sample space to show the possible ways of picking up the cards.

1st \ 2nd	A	B	C
A			
B			
C			

(c) Draw a tree diagram to show the possible ways of picking up the cards.

3 There are 3 children in the Webb family. Assuming there is an even chance of having boys and girls, complete a tree diagram to show the possible combinations of sexes for the children.

5.6e Mutually exclusive and complementary events

Two events are **mutually exclusive** if when one of them happens it stops the other happening. For example, we cannot both eat the last piece of cheesecake in the fridge. Either I eat it or you eat it (unless we are prepared to share). The events 'I eat it' and 'you eat it' are mutually exclusive.

Note: The words 'either... or...' often give a clue to events being mutually exclusive.

When events are mutually exclusive then there is no overlap.

'Choosing a Heart' and 'choosing a Spade' are mutually exclusive events because no card can be a Heart and also a Spade. 'Choosing a Heart' and 'choosing a King' are **not** mutually exclusive events because you could choose the King of Hearts.

The total sum of the probabilities of mutually exclusive events in any trial is always 1. This is because the mutually exclusive events together cover all the possible outcomes and there is no overlap.

E.g. (a) When you toss a coin it can't come down on both sides at once so 'getting a Tail' and 'getting a Head' are mutually exclusive events.

p(getting a Tail) = $\frac{1}{2}$

p(getting a Head) = $\frac{1}{2}$

So p(getting a Tail) + p(getting a Head) = $\frac{1}{2} + \frac{1}{2} = 1$

(b) In a game it is known that the probability of Rachel winning is 0·3 and the probability of Lubna winning is 0·4. What is the probability that neither Rachel nor Lubna will win?

We have three events: 'Rachel wins', 'Lubna wins' and 'neither Rachel nor Lubna wins'. Rachel and Lubna cannot both win so 'Rachel wins' and 'Lubna wins' are mutually exclusive events. Clearly if Rachel wins then it cannot also be true that 'neither Rachel nor Lubna wins' so these are mutually exclusive events. (Similarly if Lubna wins.)

So these three events are mutually exclusive and between them they cover all the possibilities.

Therefore p(Rachel wins or Lubna wins or neither) = 1

p(neither Rachel nor Lubna wins) = 1 − (0·3 + 0·4)

= 0·3

If mutually exclusive events are equally likely then the probability of each event is 1 divided by the number of events.

E.g. A bag contains five different coloured balls: a green one, a red one, a blue one, an orange one and a white one. What is the probability of choosing a ball of a particular colour?

A ball cannot be two colours at once so the events are mutually exclusive. The sum of all the probabilities must be 1.

p(green) + p(red) + p(blue) + p(orange) + p(white) = 1

But you are just as likely to choose a ball of one colour as a ball of another colour so all these probabilities are equal.

Therefore 5 × p(green) = 1

p(green) = $\frac{1}{5}$

This is the same for any colour.

Complementary events

The two events given by something 'happening' and 'not happening' are called **complementary** events. For example, the event 'throwing a 6' has the complementary event 'not throwing a 6' and 'getting a Tail' has the complementary event 'getting a Head'. If something is happening then it can't also not happen so complementary events are clearly mutually exclusive. Any two complementary events cover the whole range of possibilities because something must either happen or not happen. Using the fact that the total sum of mutually exclusive events is always 1,

p(event) + p(not event) = 1

This is very useful because it means that if you know the probability of something happening you can work out the probability of it not happening. Sometimes when you want to find the probability of an event it's far easier to calculate the probability of it not happening then take that away from 1.

E.g. If a fair die is rolled what is the probability of getting 1, 2, 3, 4 or 5?
This is the same as not getting a 6.
The probability of getting a 6 is $\frac{1}{6}$ so
p(not getting a 6) = 1 − p(getting a 6) = $1 - \frac{1}{6} = \frac{5}{6}$
Hence p(getting 1, 2, 3, 4 or 5) = $\frac{5}{6}$

?

1 (a) What is the probability of getting a 1 on a fair icosahedral die with 20 faces?
(b) Use this to calculate the probability of not getting a 1.

2 The probability of drawing a picture card (K, Q, J, A) from a pack is $\frac{4}{13}$. What is the probability of not choosing a picture card?

3 In a game of chance the probability of winning is $\frac{1}{5}$. What is the probability of losing?

4 There are red and blue counters in a bag. If the probability of choosing a red counter is $\frac{5}{8}$ what is the probability of choosing a blue counter?

5.7h Adding probabilities of mutually exclusive events

You already know that events are mutually exclusive if they can't both happen at the same time.

E.g. If Andy, Bev, Carl and Diane have a race and Bev wins, then Andy, Carl and Diane must lose. The events 'Andy wins', 'Bev wins', 'Carl wins' and 'Diane wins' are mutually exclusive.

Combining events

There are two ways to combine events.

❶ Using AND. This gives events of the form 'Andy wins **and** Bev comes second'. You are interested in events both happening.

With mutually exclusive events you know that the probability of two events both happening is always zero.

❷ Using OR. This gives events of the form 'Andy **or** Bev wins'. You are interested in at least one event happening.

With mutually exclusive events there is no overlap so **either** Andy wins **or** Bev wins **but not both.**

If you wish to work out the probability of **either** A **or** B you simply add the probabilities.

p(either A or B) = p(A) + p(B)

So p(Andy or Bev wins) = p(Andy wins) + p(Bev wins)

E.g. A family bag of crisps contains 6 packets of crisps: 3 plain, 2 cheese and onion and 1 smoky bacon. What is the probability of picking plain or smoky bacon if you choose a packet without looking?
The events are mutually exclusive because a packet of crisps cannot be plain and smoky bacon flavour at the same time.
p(choosing plain) = $\frac{3}{6}$
p(choosing smoky bacon) = $\frac{1}{6}$
p(choosing plain or smoky bacon) = $\frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

Note: Do not cancel any fractions until the final stage.

?

1 What is the probability you will get a 1 or a 2 when you roll a die?

2 A biscuit jar contains 3 chocolate, 4 cream, 2 wafer and 1 plain biscuits.
(a) Why are choosing a cream biscuit and choosing a wafer biscuit mutually exclusive?
(b) What is the probability of choosing a cream or a wafer biscuit if you choose without looking?
(c) What is the probability of choosing a chocolate or a cream biscuit if you choose without looking?

3 Jess always goes to school in one of the following ways:

	probability
by bicycle	0·4
by car	0·1
by bus	0·3
on foot	0·2

(a) Calculate the probability that she will either walk or go by bus.
(b) Calculate the probability that she will go to school in a vehicle.

Remember: When events are mutually exclusive then you add probabilities.

5.8c Calculating probabilities when independent events are combined

The probability of two independent events both happening is less than the probability for each of the individual events (unless one of the probabilities is 0 or 1).

E.g. You roll two fair dice, one green and one black. How does the probability of getting a 6 on both dice compare with the individual probabilities of getting a 6? You have already seen that when you combine two trials which involve independent events all the combinations of outcomes are possible. So there are 6 × 6 = 36 possible combinations for the scores on the dice.

The probability of getting a 6 on the black die is $\frac{1}{6}$ so the number of outcomes which give 6 on the black die is $\frac{1}{6}$ × number of outcomes = $\frac{1}{6} \times 36 = 6$.

The probability of getting a 6 on the green die is also $\frac{1}{6}$ so of these 6 outcomes the number which also give 6 on the green die is $\frac{1}{6} \times 6 = 1$.

So p(getting 6 on both dice) = 1 out of 36 = $\frac{1}{36}$

We can illustrate this by completing a probability space.

		black die					
		1	2	3	4	5	6
green die	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
	3	1,3	2,3	3,3	4,3	5,3	6,3
	4	1,4	2,4	3,4	4,4	5,4	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6

You can see that there are 36 different possible results and that only one of these gives you two 6's.

If the probability of one event is 0 that means it can't happen so the probability of it and another event both happening must also be 0.

If the probability of one event is 1 that means it must happen so the probability of both events happening will be the same as the probability for the other event.

?

1 If you have a 50p coin, a 10p coin, a 5p and a 2p in your purse what is the probability that if you take out two coins they will be a 50p coin and a 1p coin?

2 (a) Each day 1 pupil from a mixed class is chosen at random to take the register to the school office. The probability that a girl will be chosen to take the register is p. Is the probability that a girl will be chosen 2 days running
(i) the same as p? (ii) less than p? (iii) greater than p?

(b) Which of the following is the lowest?
(i) the probability that a boy will be chosen on the first day of term
(ii) the probability that a girl will be chosen on the second day of term
(iii) the probability that a boy will be chosen on the first day and a girl will be chosen on the second day

You saw above that the probability of getting a 6 on two dice at the same time is $\frac{1}{36}$. This is the same as what you get if you multiply the individual probabilities together.

p(6 on both dice) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

When two events are independent then

p(event 1 **and** event 2) = p(event 1) × p(event 2)

E.g. A bag contains 3 red counters, 4 blue counters and 5 white counters. A counter is chosen at random from the bag and then replaced and a second counter is chosen. What is the probability of choosing two red counters?

You know that when items are chosen **with replacement** then events are independent.

The first counter has been replaced so the probabilities in the second trial are the same as those in the first trial.

The probability of choosing a red counter,

$$\text{p(red)} = \frac{3}{(3+4+5)} = \frac{3}{12} = \frac{1}{4}$$

$$\text{p(red then red)} = \text{p(red)} \times \text{p(red)} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

So the probability of choosing two red counters is $\frac{1}{16}$.

?

3 If you roll a die and toss a coin what is the probability of getting a 5 and Heads?

Diagrams

One way to illustrate the probabilities for combined events is using a **probability space**.

E.g. What is the probability of taking a picture card from a pack, returning it and then taking a second picture card? Illustrate this using a probability space.

The probability of taking a picture card (A, K, Q, J) is $\frac{16}{52}$ which simplifies to $\frac{4}{13}$.

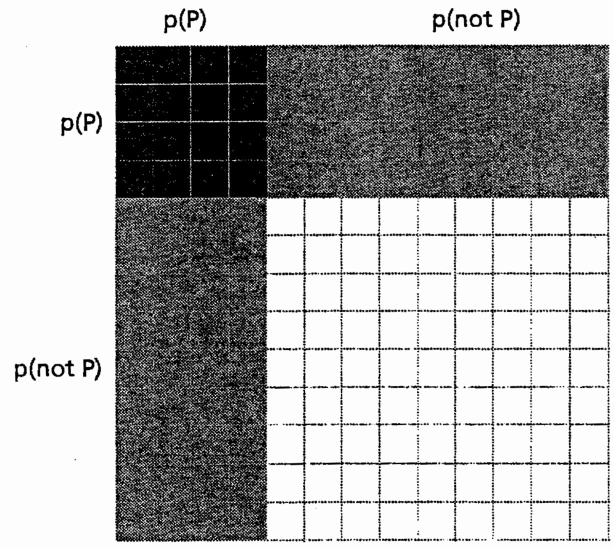
p(picture) = $\frac{4}{13}$ p(not picture) = $1 - \frac{4}{13} = \frac{9}{13}$

There are 13 different types of card, of which 4 are picture cards.

Choosing with replacement gives independent events so all combinations of outcomes are possible. Therefore choosing 2 cards gives $13 \times 13 = 169$ possibilities.

Of these, $4 \times 4 = 16$ give 2 picture cards.

So the probability of choosing 2 picture cards is $\frac{16}{169}$.



The probability space shows that out of 169 possibilities there are 16 outcomes which give 2 picture cards.

Another way to illustrate the probabilities for combined events is using a **tree diagram.**

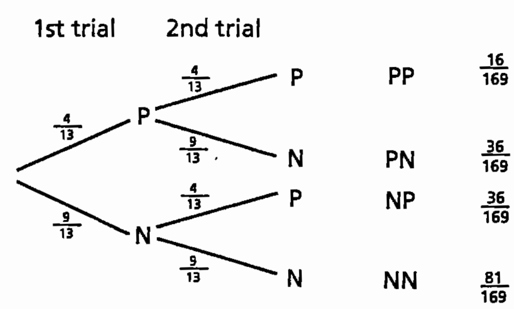
Note: The greatest problem most people find in drawing this type of diagram is in the layout. Before you start drawing work out how many branches there are going to be!

Keep the outcomes in the same order for each trial. Remember the probabilities should add up to 1 for each group of branches.

Leave any simplification of fractions to the final calculation rather than simplifying fractions on the diagram.

E.g. Draw a tree diagram to illustrate the probability of taking a picture card from a pack, returning it and then taking a second picture card.

Let P be the event 'taking a picture card' and N be the complementary event 'not taking a picture card'.



To calculate probabilities from a tree diagram you multiply along the branches.

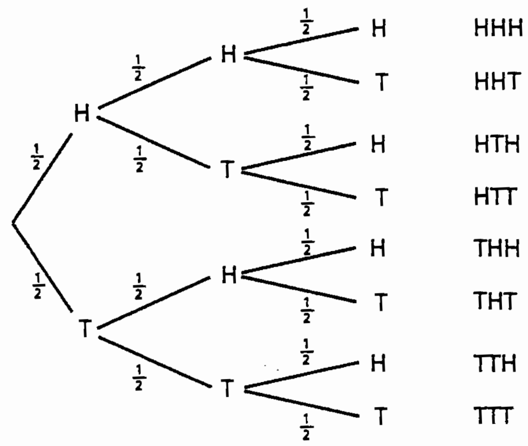
p(2 picture cards) = p(P) × p(P)

$= \frac{4}{13} \times \frac{4}{13}$

$= \frac{16}{169}$

If you go on to three trials the probability space diagram becomes impractical. But you can draw a tree diagram for any number of trials.

E.g. Draw a tree diagram to illustrate the probabilities when tossing a coin three times.



In each case the probability is $\frac{1}{2}$ so each of the possible outcomes of tossing the coin three times has probability $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ or 0·125.

4 (a) What is the probability of getting more than 4 on two successive rolls of a fair die?
(b) Draw a probability space to illustrate your answer.

5 Lisa uses her mountain bike to ride to her friend's house. She has to go through two sets of traffic lights on the way.
At the first set of lights the probability that they will be on green is 0·6.
At the second set of traffic lights the probability that they will be on green is 0·5.
(a) Calculate the probability that the first set of traffic lights will not be green.
(b) Calculate the probability that the second set of traffic lights will not be green.
(c) Draw a tree diagram to illustrate the probabilities of the two sets of lights being green or not green.
(d) Calculate the probability that Lisa will be able to ride to her friend's house without having to stop at any traffic lights.

1 Say which of the following you would study by sampling. Give reasons for your answers. What are the factors you would take into account in the cases you would test by sampling?

(a) The average life of a battery.
(b) The type of cat food most cats prefer.
(c) The top ten singles sold last week.
(d) The cables on a lift.

Sometimes you have no information about the characteristics of a population. Then you should choose a sample in which all items are equally likely to be chosen. This is called a **random sample.**

To ensure the sample is random and as accurate as possible it should be repeated a number of times. The random samples can then be averaged.

2 To carry out the following experiment you will need 100 counters or other objects which are the same except for their colour. Put the counters in a bag. Choose a colour.

Experiment: Take 10 counters without looking and record the number of your chosen colour. Multiply the result by 10 to predict how many of your chosen colour are in the bag. Replace the counters.

Repeat the experiment 10 times, recording your results in a table.

Average out the results of the 10 experiments to find the mean number of counters you picked of your chosen colour.

Multiply the mean by 10 to get an estimate for the total number of counters of that colour.

Empty the bag and count how many counters of your chosen colour there are.

Compare this with your first prediction and the final mean estimate.

You should have found that the results obtained by repeating the trial were more accurate because they were based on more information.

The technique of random sampling can be used to estimate the size of a population.

E.g. The crested newt is an endangered species. Conservationists want to find out the number of crested newts in a pond. To do this they catch 10 newts and mark them in a harmless way.

The newts are released back into the pond and the next day 10 more newts are caught. Of the 10 newts 2 are found to have been marked the previous day. The result on the 2nd day is that 2 out of a sample of 10 newts are marked.

Applying the results of this sample to the whole population means that 20% of the population should be marked. But the conservationists know that 10 newts are marked. So 20% of the population is about 10 newts.

Let the size of the total population be P.

$$\frac{20}{100} \times P \approx 10$$

$$\frac{1}{5} \times P \approx 10$$

$$P \approx 50$$

So there are about 50 newts in the pond.

5.9d Conditional probabilities

If the probability of an event happening depends on whether another event took place or not, then we say the events are **dependent**. The probability is said to be **conditional**.

Whenever the outcome of one trial affects the possible outcomes of a second trial then events are dependent. If two items are chosen **without replacement** then the item chosen first cannot be chosen again so the probabilities for the second choice are conditional on what was chosen first.

E.g. In a biscuit barrel there are 5 chocolate biscuits, 4 plain and 6 wafers.

(a) A biscuit is taken at random and eaten. Write down the probability of choosing each type of biscuit.

(b) A second biscuit is taken. What is the probability that both biscuits were chocolate?

(a) The total number of biscuits is 5 + 4 + 6 = 15. The words 'at random' tell you that each biscuit is equally likely to be chosen so

p(choosing chocolate) = $\frac{5}{15}$
p(choosing plain) = $\frac{4}{15}$
p(choosing wafer) = $\frac{6}{15}$

(b) If the first biscuit is eaten then it cannot be replaced! So the probabilities for the second choice are conditional on which biscuit was chosen first.

The probability of choosing a chocolate biscuit in the first trial was $\frac{5}{15}$.

If that biscuit is eaten then there will only be 4 chocolate biscuits left and only 14 left in the barrel. This means that on the second occasion the probability of choosing a chocolate biscuit is $\frac{4}{14}$. This is conditional on having chosen chocolate in the first trial.

We write this as p(chocolate 2nd choice given chocolate 1st choice) or p(chocolate/chocolate).

The probability of choosing 2 chocolate biscuits is

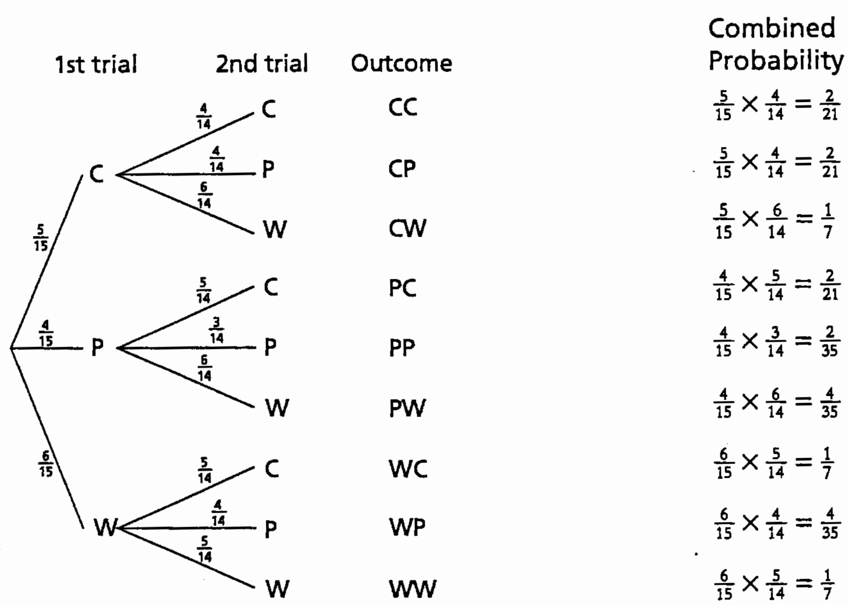
p(chocolate 1st choice) × p(chocolate 2nd choice given chocolate 1st choice)

$p(C) \times p(C/C) = \frac{5}{15} \times \frac{4}{14}$

$= \frac{2}{21}$

The tree diagram looks like this. To get the combined probabilities you multiply along the branches. Do your simplifying in the calculation and not on the diagram.

Remember: In the 2nd trial the number you are choosing from is 1 less.



With conditional probability the probability of A followed by B is written

p(A and B) =p(A) × p(B given A) or p(A and B) = p(A) × p(B/A)

1 In a class there are 14 girls and 13 boys. Two pupils are to be chosen at random to give books out. Draw a tree diagram to illustrate the probabilities.

2 Mark has bought a packet of 9 hyacinth bulbs which all look the same. 4 of the bulbs will have red flowers, 3 blue and 2 white.
(a) If Mark chooses 2 bulbs from the packet calculate the probability that they will both have blue flowers.
(b) Calculate the probability that the first bulb chosen will have white flowers and the second will have red flowers.

An alternative to drawing a tree diagram to represent dependent probabilities is to use a sample space. This shows all the possible outcomes. If a combination is not possible then you blank out that square.

E.g. A teacher needs two pupils to help on parents' evening. Two pupils are to be chosen at random from Alice, Brian, Claire, Derek and Elaine.
Illustrate the possible combinations for choosing two pupils from these five.

		Followed by				
		A	B	C	D	E
	A		AB	AC	AD	AE
First	B	BA		BC	BD	BE
choice	C	CA	CB		CD	CE
	D	DA	DB	DC		DE
	E	EA	EB	EC	ED	

Note: Since you need two pupils this is without replacement.

You cannot choose Alice and then Alice again so that choice can be blanked out.

The number of possibilities for the first choice is 5 but the number of possibilities for the second choice is 1 less. So the total number of possible combinations is $5 \times 4 = 20$.

3 In a packet of yoghurts there are 4 different varieties: strawberry, raspberry, lemon and blackcurrant. Mr Smith has one today and one tomorrow.
Illustrate Mr Smith's choices using a sample space.

5.10d Calculating the probability for any two events

You know how to calculate probabilities when events are independent or dependent or mutually exclusive. You now need to be able to apply what you know about probability to be able to calculate the probability for any two events.

There are a number of questions you should ask yourself before you start calculating the probabilities.

Step 1 What event are you looking for the probability of?

Step 2 Can you use the fact that p(event) = 1 − p(not event) to save you work? (You may already have calculated p(not event).)

Step 3 Is the event a combination of two or more mutually exclusive events whose probabilities you can add?

Step 4 Are these events themselves combinations of two other events, either dependent or independent?

Here is an example to show how useful these steps can be.

E.g. Find the probability of getting a 5 or Heads (or both) if you toss a coin and roll a fair die.

Without using the steps above:

It is possible to get a 5 and Heads at the same time so the events 'getting a 5' and 'getting Heads' are not mutually exclusive. In fact the event 'getting a 5 or Heads or both' is a combination of **three** mutually exclusive events:

'5 and not Heads', 'Heads and not 5' and '5 and Heads'

p(5 or H or both) = p(5 and not H) + p(H and not 5) + p(5 and H)

The result you get on the coin will have no effect on the result on the die (and vice versa) so the individual events involved are independent and you can multiply the probabilities.

Hence p(5 and not H) = p(5) × p(not H)

$= \frac{1}{6} \times \frac{1}{2}$

$= \frac{1}{12}$

p(H and not 5) = p(H) × p(not 5)

= p(H) × [1 − p(5)]

$= \frac{1}{2} \times \frac{5}{6}$

$= \frac{5}{12}$

p(5 and H) = p(5) × p(H)

$= \frac{1}{6} \times \frac{1}{2}$

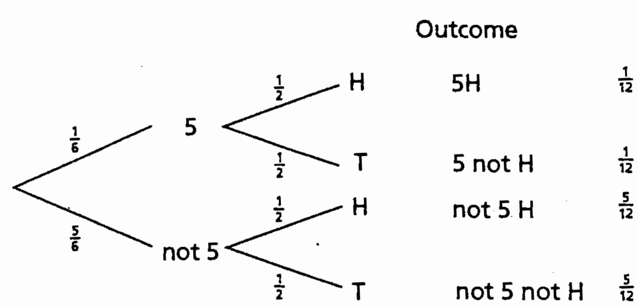
$= \frac{1}{12}$

Adding the probabilities of these mutually exclusive events gives

p(5 or H or both) $= \frac{1}{12} + \frac{5}{12} + \frac{1}{12}$

$= \frac{7}{12}$

This is easier to see if you draw a tree diagram.



It is easier still if you use the steps given on the previous page.

Step 1 event = 5 or Heads or both

Step 2 p(5 or Heads or both) = 1 − p(neither 5 nor Heads)

Step 4 neither 5 nor Heads = not 5 **and** not Heads

'not 5' and 'not Heads' are two independent events so multiply the probabilities:

p(not 5 and not Heads) = p(not 5) × p(not Heads)

$= \frac{5}{6} \times \frac{1}{2}$

$= \frac{5}{12}$

So p(5 or Heads or both) $= 1 - \frac{5}{12}$

$= \frac{7}{12}$

Remember: Whenever you can, use the fact that p(event) = 1 − p(not event) to make things easier.

You can see from this example that sometimes it's much quicker not to draw a diagram.

E.g. A bag contains 2 red counters, 7 orange counters and 3 yellow counters. A counter is chosen without looking and then replaced. A second counter is then chosen. Calculate the probability that the two counters will be the same colour.

Step 3 p(2 same colour) = p(2 red) + p(2 orange) + p(2 yellow)

Events are independent because the counters are replaced. The probabilities are the same each time a choice is made.

Step 4 p(2 same colour) = p(red) × p(red) + p(orange) × p(orange) + p(yellow) × p(yellow)

$= \frac{2}{12} \times \frac{2}{12} + \frac{7}{12} \times \frac{7}{12} + \frac{3}{12} \times \frac{3}{12}$

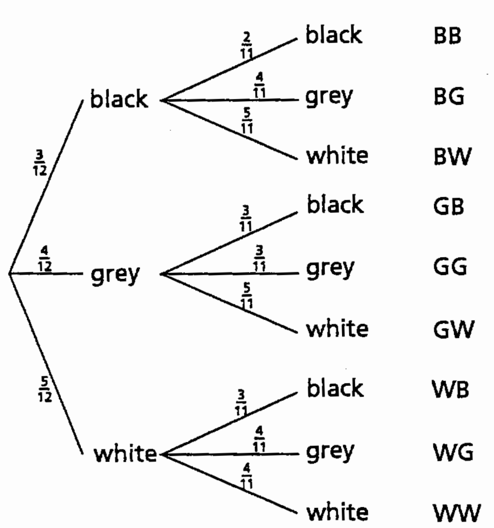
$= \frac{4}{144} + \frac{49}{144} + \frac{9}{144}$

$= \frac{31}{72}$

1. A card is drawn at random from a pack of 52 and then replaced. A second card is then chosen. Calculate the probability that:
   (a) both cards will be picture cards. (These include Aces.)
   (b) neither card will be a picture card.
   (c) at least one card will be a picture card.

When items are chosen without replacement then the probabilities are different for the second trial (conditional probability). In this case it may well be easier if you draw a diagram, especially if there are several parts to the question. Once you've drawn the diagram you can read off the probabilities required to answer the various parts of the question.

E.g. Simon's drawer contains 3 black socks, 4 grey socks and 5 white socks. Simon pulls two socks from the drawer without looking.
(a) What is the probability he chooses 2 socks of the same colour?
(b) What is the probability he chooses first a grey and then a black sock?
(c) What is the probability that one sock will be grey and the other black?
Simon is choosing without replacement so the events are dependent.



(a) p(2 same colour) = p(2 black) + p(2 grey) + p(2 white)

$= \frac{3}{12} \times \frac{2}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{5}{12} \times \frac{4}{11}$

$= \frac{19}{66}$

(b) p(grey then black) = p(grey) × p(black/grey)

$= \frac{4}{12} \times \frac{3}{11}$

$= \frac{1}{11}$

(c) There are two ways to get a grey sock and a black sock: first grey then black or first black then grey.

p(grey and black) = p(grey then black) + p(black then grey)

From (b), p(grey then black) = $\frac{1}{11}$

$= \frac{1}{11}$ + p(black) × p(grey/black)

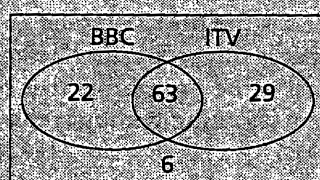
$= \frac{1}{11} + \frac{3}{12} \times \frac{4}{11}$

$= \frac{2}{11}$

2 A car dealer has 6 red cars, 5 blue cars and 2 white cars. He sells one car in the morning and a second in the afternoon.
(a) Draw a tree diagram to illustrate the probabilities.
(b) What is the probability that
(i) the cars are both blue?
(ii) the cars are both the same colour?
(iii) at least one car is blue?

3 Sajpal and Michael carry out a survey of 120 Year 7 pupils to find out whether they watch BBC or ITV television, both or neither.
The results are as follows:

BBC	22
ITV	29
neither	6
both	63



(a) If a child is chosen at random, what is the probability that he or she watches at least some BBC programmes?
(b) Given that Sarah watches ITV, what is the probability that she also watches BBC?
(c) If two children are chosen at random, what is the probability that they both watch television but not the same channel?

3.10b Quadratic equations

You have already seen how to solve quadratic equations by drawing graphs and finding points of intersection. This unit covers various algebraic methods for solving quadratic equations.

Factorisation

You already know how to factorise quadratic expressions into two brackets of the form $(ax + b)(cx + d)$.
So given a quadratic equation you can write it in the form

$$(ax + b)(cx + d) = 0$$

When two numbers multiply together to give zero then at least one of the numbers must itself equal zero.

Therefore $(ax + b) = 0$ or $(cx + d) = 0$ or both.

If $(ax + b) = 0$ then $x = -\frac{b}{a}$ and if $(cx + d) = 0$ then $x = -\frac{d}{c}$

so the solutions (roots) of the quadratic equation are $x = -\frac{b}{a}$ and $x = -\frac{d}{c}$.

E.g. Solve the equation $6x^2 + 5x - 6 = 0$ by factorising.

First factorise $6x^2 + 5x - 6$.
You need to find two factors of 6 which combine with another two factors of 6 to give 5. Because the number term is negative you are looking for the **difference** to equal 5.

Hint: Start by trying the factors which are closest together.

The closest factors of 6 are 2 and 3. Try the factor pairs (2, 3) and (2, 3).
$3 \times 3 - 2 \times 2 = 5$ which gives the result you want.

One factor pair gives the coefficients of x so start with
$(2x \quad)(3x \quad)$
The other factor pair goes at the ends of the brackets. To get 5 you need to multiply the 3's together and the 2's together so this tells you which bracket to put each number in.
$(2x \quad 3)(3x \quad 2)$
Because the number term in the original expression is negative the signs in the brackets will be different. To get 5 you subtracted the 2×2 so the – goes in front of the 2.
$(2x + 3)(3x - 2)$

Check: $(2x + 3)(3x - 2) = 6x^2 - 4x + 9x - 6 = 6x^2 + 5x - 6$

So $6x^2 + 5x - 6 = (2x + 3)(3x - 2)$
Putting $(2x + 3)(3x - 2) = 0$
gives $(2x + 3) = 0$ or $(3x - 2) = 0$
So the solutions are $x = -\frac{3}{2}$ and $x = \frac{2}{3}$.

Remember: Always check your answer by multiplying the brackets.

?

1 Solve the following quadratics by factorising:
(a) $8x^2 + 16x + 6 = 0$ (b) $9x^2 + 27x + 20 = 0$ (c) $6x^2 - 40x - 14 = 0$

Not all quadratics can be factorised into brackets with integer (whole number) coefficients (e.g. $x^2 - 7$) so we need an alternative method.

Hint: If a question asks you to solve a quadratic to a given number of decimal places then you can't do it by factorising!

Completing the square

One method which works for all quadratic equations (unless they don't have any solutions – see below) is completing the square. This is a way of getting x to appear only once in the equation so that you can then simply rearrange it to find the values of x.

E.g. (a) Solve the quadratic $x^2 - 7x - 12 = 0$ to 2 decimal places.

First get the terms involving x on one side and the number on the other.

$$x^2 - 7x = 12$$

Then write the expression on the left hand side (LHS) in the form of a square. To do this you divide the coefficient of x (the number in front of x) by 2.

$$(x - \tfrac{7}{2})^2$$

Expanding this bracket gives $x^2 - 7x + \frac{49}{4}$ or $x^2 - 7x + 12{\cdot}25$

So by adding $12{\cdot}25$ to the LHS you can complete the square. To preserve the equation you must also add $12{\cdot}25$ to the RHS.

$$\begin{aligned} x^2 - 7x &= 12 \\ x^2 - 7x + 12{\cdot}25 &= 12 + 12{\cdot}25 \\ (x - 3{\cdot}5)^2 &= 24{\cdot}25 \end{aligned}$$

Taking square roots,

$$\begin{aligned} x - 3{\cdot}5 &= \pm\sqrt{24\cdot25} \\ x &= 3{\cdot}5 \pm\sqrt{24\cdot25} \end{aligned}$$

To 2 decimal places the solutions are $x = 8{\cdot}42$ and $x = {}^{-}1{\cdot}42$.

(b) Solve the quadratic $3x^2 + 2x - 4 = 0$ to 2 decimal places.
First get all the terms involving x on the left hand side (LHS).

$$3x^2 + 2x = 4$$

Then divide through by the coefficient of x^2.

$$x^2 + \tfrac{2}{3}x = \tfrac{4}{3}$$

Divide the coefficient of x by 2 then square it and add to both sides.

$$x^2 + \tfrac{2}{3}x + \tfrac{1}{9} = \tfrac{4}{3} + \tfrac{1}{9}$$

This can now be factorised.

$$\left(x + \tfrac{1}{3}\right)^2 = \tfrac{13}{9}$$

Taking the square root of both sides

$$x + \tfrac{1}{3} = \pm\sqrt{\tfrac{13}{9}}$$

To 2 decimal places the solutions are $x = 0{\cdot}87$ and $x = {}^{-}1{\cdot}54$.

? 2 Solve the following equations to 2 decimal places by completing the square:
(a) $x^2 + 3x - 5 = 0$ (b) $x^2 - 2x - 34 = 0$

The formula

You can apply the same process of completing the square to the equation $ax^2 + bx + c = 0$. (Try doing this yourself.)

This gives the following formula for the solutions of a quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

E.g. Solve the quadratic $3x^2 + 2x - 4 = 0$ to 2 decimal places.

Comparing with $ax^2 + bx + c = 0$ you have $a = 3$, $b = 2$ and $c = {}^{-}4$.
Substituting into the formula

$$x = \frac{{}^{-}2 \pm \sqrt{2^2 - 4 \times 3 \times {}^{-}4}}{2 \times 3}$$

$$= \frac{{}^{-}2 \pm \sqrt{4 + 48}}{6}$$

$$= \frac{{}^{-}2 \pm \sqrt{52}}{6}$$

To 2 decimal places the roots are $x = 0{\cdot}87$ and $x = {}^{-}1{\cdot}54$.

?

3 Solve the following quadratics to 2 decimal places using the formula:
(a) $2x^2 - 3x - 7 = 0$ (b) $3x^2 - 5x - 1 = 0$

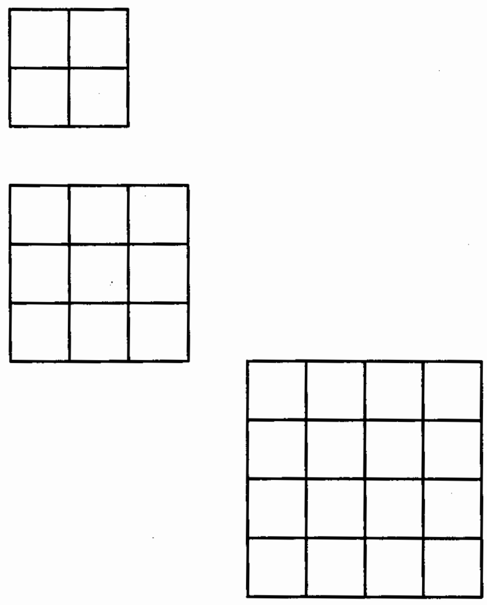
Appendix 4

Nozipho's journey to school

Nozipho lives at Lobamba. She goes to school at St Theresa's in Manzini. Every morning she walks for 10 minutes from her house to a bus stop. She waits 5 minutes before a minibus arrives. The minibus takes 5 minutes to load then drives off towards Manzini. It arrives at Mahlanya after 10 minutes then stops for 10 minutes unloading and loading. It leaves Mahlanya and arrives at Matsapha after 15 minutes there it unloads and loads for 10 minutes. It arrives in Manzini after 10 minutes. Nozipho gets off the minibus 5 minutes after it stops, walks 12 minutes to school. School starts at 7.30am and finishes at 3.30pm. The table shows the various distances on the way to school.

From	Destination	Distance
Home	Bus stop	0.5km
Bus stop	Mahlanya	5km
Mahlanya	Matsapha	10km
Matsapha	Manzini	10km
Manzini	School	1.5km

Appendix 5



(i) The first big square is covered by 4 unit squares.
(ii) 4 of these unit squares (touchers) touch the edges of the big square.
(iii) The distance around the big square is 8 units.

(i) The second big square is covered by 9 unit squares.
(ii) 8 of these unit squares touch the edges of the big square.
(iii) The distance around the big square is 12 units.

(i) The third big square is covered by …….. unit squares.
(ii) ………. Of these unit squares touch the edges of the big square.
(iii) The distance around the big square is ……… units.

(a) Copy and complete the spaces for the third big square.
(b) Draw a forth big square and write three statements on it as for the others.
(c) Draw and complete a table that you would use to predict answers (i)-(iii) for any diagram like these.
(d) Write a general formula for predicting each of the answers (i) –(iii) From the size of the square.