UNIVERSITY OF SWAZILAND
FACULTY OF EDUCATION
FINAL EXAMINATION PAPER 2010

TITLE OF PAPER: CURRICULUM STUDIES IN MATHEMATICS

COURSE CODE: EDC 381

PROGRAMME: B.ED 3 & PGCE

APPENDICES: SGCSE MATHEMATICS SYLLABUS & TRIGONOMETRY CHAPTERS

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS: ANSWER QUESTION 5 AND ANY OTHER **THREE** QUESTIONS. EACH QUESTION IS WORTH 25 MARKS.

This paper contains 3 pages including this one

Question 1

(a) Explain in your own words each of the following phrases in relation to testing:

(i) Marker unreliability [5]

(ii) Administrative unreliability [5]

(b) The following question was in the 2006 paper 4 IGCSE mathematics examination.

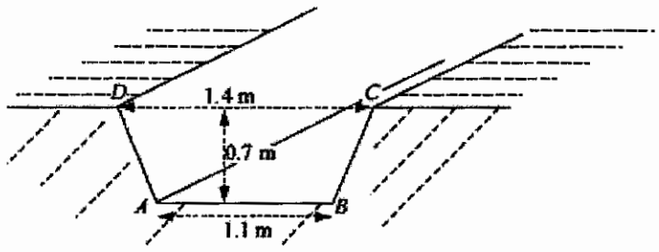
(i) Work out the answer to this question and prepare an extensive marking guide for it using symbols learnt in this course [10]

(ii) Analyse the question and say why it is a good or a bad question [5]

Examination question

3 Workmen dig a trench in level ground.

NOT TO SCALE



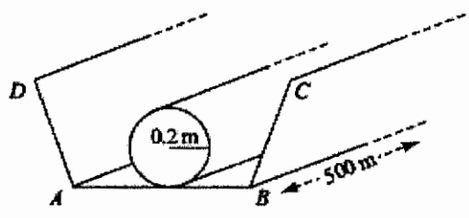
(a) The cross-section of the trench is a trapezium $ABCD$ with parallel sides of length 1.1 m and 1.4 m and a vertical height of 0.7 m.
Calculate the area of the trapezium. [2]

(b) The trench is 500 m long.
Calculate the volume of soil removed. [2]

(c) One cubic metre of soil has a mass of 4.8 tonnes.
Calculate the mass of soil removed, giving your answer in tonnes and in standard form. [2]

(d) Change your answer to part (c) into grams. [1]

NOT TO SCALE



(e) The workmen put a cylindrical pipe, radius 0.2 m and length 500 m, along the bottom of the trench, as shown in the diagram.
Calculate the volume of the cylindrical pipe. [2]

(f) The trench is then refilled with soil.
Calculate the volume of soil put back into the trench as a percentage of the original amount of soil removed. [3]

Question 2

Section 14.7 of the SGCSE mathematics syllabus reads "solve trigonometric problems in 3-dimensions including angle between a line and a plane" In trying to make sense of this section you found the material in ***appendix 2.***

(a) Use the material to highlight what the learners need to learn in this section [15]
(b) Write brief notes on how you intend to facilitate learning of this section [10]

Question 3

Write an organizational plan for the mathematics department of the school where you are the head. Introduce the reader to the school such that he/she is aware of the type of school the type of learners and the qualifications of teachers you have to work with [6]. The plan should show clearly how you intend to organize the following for the effective teaching and learning of mathematics:

(i) Teaching learning materials including text books and reference books [5]
(ii) Teachers [5]
(iii) Learners [5]

Finally write a conclusion on departmental organization [4]

Question 4

Questions 1 & 2 below are from a junior certificate multiple choice examination.

(a) Study each question to: identify the key, analyse each distractor and decide if it is plausible. In other words find out how each distractor might arise. I each case [18]
(b) For each question establish if the key can not be obtained by using an incorrect method [7]

The Questions

1. The bearing of P from Q is 300°. The bearing of Q from P is:
   A 40° B 60° C 120° D 300° E 240°
2. The total surface area of a cube of side length 6 cm is:
   A $36cm^2$ B $108\ cm^2$ C $144\ cm^2$ D $216\ cm^2$ E $180\ cm^2$

Question 5

Using what you learnt on essay writing, write an essay entitled "The best approach to the SGCSE mathematics syllabus is ability grouping" Your essay should indicate a clear understanding of the SGCSE mathematics syllabus, its implications for the learners, your definition of ability grouping [25].

APPENDICES

21 PAGES

Swaziland General Certificate of Secondary Education

Mathematics

2009-2010 Syllabus

Appendix 1

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

CONTENTS

Developed in collaboration with the University of Cambridge International Examinations (CIE), part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

SWAZILAND GENERAL CERTIFICATE OF SECONDARY EDUCATION

Broad Guidelines

The Ministry of Education is committed, in accordance with the National Policy Statement on Education, to provide a Curriculum and Assessment System (Form 4 and Form 5) so that at the completion of secondary education, learners will

- be equipped to meet the changing needs of the Nation, and
- have attained internationally acceptable standards.

Swaziland's National Education Policy Directives

SGCSE syllabuses for studies in Form 4 and Form 5 will individually, and collectively, enable learners to develop **essential skills** and provide a broad **learning experience** which

- inculcates values and attitudes as well as knowledge and understanding,
- encourages respect for human rights and freedom of speech,
- respects the values and beliefs of others, relating to issues of gender, culture and religion.
- develops desirable attitudes and behaviour towards the environment,
- provides insight and understanding of global issues which affect quality of life in Swaziland and elsewhere, e.g., the AIDS pandemic; global warming; maldistribution of wealth; and technological advances.

The National Curriculum for Form 4 and Form 5

Learners will be given opportunities to develop **essential skills** which will overlap across the entire range of subjects studied. These skills are listed below.

- Communication and language skills
- Numeracy skills: mathematical ideas, techniques and applications
- Problem-solving skills
- Technological awareness and applications
- Critical thinking skills
- Work and study skills
- Independent learning
- Working with others

To develop these skills, learners must offer **four compulsory subjects** and at least **three elective subjects** chosen from one or more Field of Study.

Compulsory Subjects
- SiSwati – either First Language or Second Language
- English Language
- Mathematics
- Science

Fields of Study
- Agriculture Field of Study
- Business Studies Field of Study
- Home Economics Field of Study
- Social Sciences and Humanities Field of Study
- Technical Field of Study

Appendix 1

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

INTRODUCTION

The Swaziland General Certificate of Secondary Education (SGCSE) syllabuses are designed as two-year courses for examination in Form 5. The syllabus assumes that learners have acquired knowledge, understanding and skills during their study of Mathematics at Junior Secondary Level. The curriculum content of the syllabus is arranged into topics covering four areas: Number; Shape, Position and Space; Algebra; and Data Handling, but it is treated throughout in a holistic way. It is intended to promote imaginative and innovative styles of teaching and learning so that the course is enjoyable for all learners, and is designed to assess what learners know, understand and can do. As such, it will enable learners to progress to higher-level courses of mathematical studies.

Learners will follow the Core plus the Extended Curriculum. The Core Curriculum is examined by two written papers. Those candidates who are expected to achieve a Grade C or above will be entered for a third written paper. The papers are described in the Scheme of Assessment.

All SGCSE syllabuses follow a general pattern. The main sections are:
Aims
Assessment Objectives
Assessment
Curriculum Content

Mathematics falls into the Mathematics Compulsory Subjects Group.

AIMS

The aims of the curriculum are the same for all learners. The aims are set out below and describe the educational purpose of a course in Mathematics for the SGCSE Examination. They are not listed in order of priority.

The aims are to enable students to:

1. develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment;
2. read mathematics and write and talk about the subject in a variety of ways;
3. develop a feel for number, carry out calculations and understand the significance of the results obtained;
4. apply mathematics in everyday situations and develop an understanding of the part which mathematics plays in the world around them;
5. solve problems, present the solution clearly, check and interpret the results;
6. develop an understanding of mathematical principles;
7. recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem;
8. use mathematics as a means of communication with emphasis on the use of clear expression;
9. develop the ability to apply mathematics in other subjects, particularly science and technology;
10. develop the abilities to reason logically, to generalise and to prove;
11. appreciate patterns and relationships in mathematics;
12. produce and appreciate imaginative and creative work arising from mathematical ideas;
13. develop their mathematical abilities by considering problems and conducting individual and cooperative enquiry and experiment, including extended pieces of work of a practical and investigative kind;
14. appreciate the interdependence of different branches of mathematics;
15. acquire a foundation appropriate to their further study and of other disciplines.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

ASSESSMENT OBJECTIVES

There is a single Assessment Objective in Mathematics

TECHNIQUE WITH APPLICATION

A description of the assessment objective follows.
Learners should be able to:

1. organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms;
2. perform calculations by suitable methods;
3. use an electronic calculator;
4. understand systems of measurement in everyday use and make use of them in the solution of problems;
5. estimate, approximate and work to degrees of accuracy appropriate to the context;
6. use mathematical and other instruments to measure and to draw to an acceptable degree of accuracy;
7. interpret, transform and make appropriate use of mathematical statements expressed in words or symbols;
8. recognise and use spatial relationships in two and three dimensions, particularly in solving problems;
9. recall, apply and interpret mathematical knowledge in the context of everyday situations;
10. make logical deductions from given mathematical data;
11. recognise patterns and structures in a variety of situations, and form generalisations;
12. respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form;
13. analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution;
14. apply combinations of mathematical skills and techniques in problem solving;
15. set out mathematical work, including the solution of problems, in a logical and clear form using appropriate symbols and terminology.

Appendix 1

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

Specification Grid

Objective Number	Short-answer questions	Structured/longer answer questions	CORE	EXTENDED
1 and 2	✓	✓	✓	✓
3		✓	✓	✓
4 to 8	✓	✓	✓	✓
9	✓	✓	✓	
10	✓	✓	✓	✓
11		✓	✓	✓
12		✓		✓
13		✓		✓
14	✓	✓	✓	✓
15		✓		✓

The grid above is for general guidance only and illustrates where particular objectives might receive more emphasis. Ticks are placed in the grid only where there is likely to be emphasis although the objective may also be met in other areas. There is no rigid emphasis between particular assessment objectives and individual examination components; the objectives may be assessed in any question. The components of the scheme will differ in emphasis placed on various objectives.

The short-answer questions fulfill a particularly important function in ensuring Core Curriculum coverage and allowing the testing of knowledge, understanding and manipulative skills, while greater emphasis is placed on applications to the processes of problem solving in the structured/longer answer questions.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

ASSESSMENT

Scheme of Assessment

All candidates must enter for two papers. These will be Paper 1 and Paper 2. In addition, those candidates who are expected to achieve a Grade C or above should be entered for Paper 3.

Core Curriculum Grades C to G available	**Extended Curriculum** Grades A* to G available
Paper 1 (1 hour and 30 minutes) **Compulsory** short-answer paper consisting of 60 marks, with questions designed to discriminate between grades C to G. The questions will be based on the Core Curriculum. This paper will be weighted at 40% of the final total available marks.	
Paper 2 (2 hours) **Compulsory** structured/longer answer paper consisting of 90 marks, with questions designed to discriminate between grades C to G. The questions will be based on the Core Curriculum. This paper will be weighted at 60% of the final total available marks.	
[redacted]	**Paper 3** (2 hours) **Compulsory** structured/longer answer paper consisting of 100 marks, with questions designed to discriminate between grades A to C. The questions will contain material from the Extended Curriculum as well as the Core. This paper will be weighted at 100% of the final total available marks.

Notes:

1. Use of an Electronic Calculator and Mathematical Tables:
   (i) All candidates should be able to use an electronic calculator efficiently and apply it appropriately to the required degree of accuracy.
   (ii) The syllabus assumes that candidates will be in possession of a scientific electronic calculator for Papers 2 and 3. Algebraic or graphical calculators are not permitted. Three significant figures will be required in answers except where otherwise stated.
   (iii) The use of electronic calculators or mathematical tables is prohibited in Paper 1.
2. Use of Mathematical Instruments:
   Apart from the usual mathematical instruments, candidates may use flexicurves in this examination.
3. Candidates are encouraged to use the value of pi (π) from their calculators if their calculator provides this. Otherwise, they should use the value of π as given in the question or on the front page of the question paper.
4. Tracing paper may be used as an optional additional material for each of the written papers.

Appendix 1

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

Weighting of papers

Paper	Weighting Core Curriculum candidates (Papers 1 and 2 only)	Weighting Extended Curriculum candidates (Papers 1, 2 and 3)*
1	40%	40%
2	60%	60%
3	Not taken by Core Curriculum candidates	100%

Note:
* Extended Curriculum candidates write all three papers. They will be assessed on their performance on Paper 3 first to see if they have achieved A* to C. If they have not done so, their grade will be assessed on the basis of their performance on Papers 1 and 2.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations.

CURRICULUM CONTENT

Learners will follow the Core plus the Extended Curriculum. The curriculum content that follows is divided into topics covering four areas: Number; Shape, Position and Space; Algebra; and Data Handling. An indication of the area covered by a topic is provided in brackets after the topic heading. The table below shows the approximate weighting of these areas in each of the components of the examination.

Paper	Number	Shape Position Space	Algebra	Data Handling
1	30%	35%	25%	10%
2	20%	35%	30%	15%
3	10%	35%	35%	20%

As well as demonstrating skill in the following techniques, candidates will be expected to apply them in the solution of problems.

Appropriate teaching time for the Mathematics syllabus should be equivalent to seven (7) periods of forty (40) minutes each over a period of sixty (60) weeks/cycles.

CORE	EXTENDED
1. Types of Numbers and their Sequences, Sets and Set Notation and Language *[Topic Area: Number]*	
All learners should be able to: 1.1 Identify sets of primes, multiples, factors, squares, cubes in natural numbers. *Identify includes listing and describing.* 1.2 Express natural numbers as products of their prime factors. 1.3 Identify common multiples and common factors (e.g., LCM and HCF). 1.4 List directed numbers. 1.5 Identify sets and subsets of real numbers. (i.e., natural numbers, primes, factors, rational and irrational numbers) in a sequence. 1.6 Find missing numbers in a sequence of: (i) composite numbers (ii) triangle numbers (iii) rectangle or square numbers (iv) Pascal's triangle. 1.7 Find rules for simple number patterns. 1.8 For 2 sets and a universal set, draw Venn diagrams and use the language and notation of sets (i.e., subsets, union, intersection complement number of elements).	1.10 List and describe elements of the real number system. 1.11 Complete and generate number patterns. 1.12 Generalise to simple algebraic statements. 1.13 Form an equation by generalisation (*n*th term) of a given sequence. 1.14 List and describe elements and use set symbols. 1.15 For more than 2 sets and a universal set, draw Venn diagrams and use the language and notation of sets (i.e., subsets, union, intersection complement number of elements).

1.9 Describe and use set symbols: { } "... is a set of ..." $\in$ "... is an element of..." $\notin$ "...is not an element of..." $\varnothing$ the empty set $\cap$ intersection of $\cup$ union of $\subset$ proper subset of $\subseteq$ is a subset of $\not\subset$ is not a proper subset of A′ complement of set A $\mathfrak{E}$ universal set n(*A*) number of elements in set A	1.16 Use set builder notation to describe sets.
2. Place Value, Estimation and Limits of Accuracy *[Topic Area: Number]*	
All learners should be able to: 2.1 Round off to specified level of accuracy e.g., nearest 50. 2.2 Estimate quantities (length, volume, capacity, area population etc). 2.3 Approximate to a given number of significant figures or decimal places and round off answers to reasonable accuracy in the context of a given problem. 2.4 Approximate numbers to a given place value and/or decimal place.	2.5 Find the upper and lower bounds for a given specified accuracy e.g., population, time, distance, speed, etc. 2.6 Use the determined limits of measurements to calculate the limits of a perimeter of a given figure. 2.7 Use the determined limits of measurements to calculate the limits of area of a given figure.
3. Operations *[Topic Area: Number]*	
All learners should be able to: 3.1 Add and subtract fractions. 3.2 Multiply and divide fractions by whole numbers and fractions. 3.3 Convert fractions to: (i) their equivalent fractions (ii) improper fractions and vice-versa. 3.4 Write fractions in order of size. 3.5 Add and subtract decimals. 3.6 Divide and multiply decimals by powers of 10. 3.7 Multiply and divide decimals by (i) a whole number (ii) decimals. 3.8 Simplify numerical expressions involving mixed operations. 3.9 Apply correct order of operations. 3.10 Convert fractions into decimals and percentages, and decimals into fractions and percentages. 3.11 Convert percentages into fractions and decimals.	

3.12 Find squares, cubes, square roots and cube roots of numbers. 3.13 Understand the four operations of directed numbers. 3.14 Apply appropriate checks of accuracy. 3.15 Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, <, >, ≤, ≥.	
4. Percentages *[Topic Area: Number]*	
All learners should be able to: 4.1 Calculate a percentage of a given quantity or constant. 4.2 Calculate one quantity as a percentage of another. 4.3 Calculate (i) the percentage change given the new and original value, (ii) the new value given the original and percentage change. 4.4 Calculate: (i) the percentage profit or loss given the buying and the selling price (ii) the selling price, given the buying price and the percentage loss and profit. 4.5 Calculate the simple interest due to a customer after a certain period of time, given the percentage interest per annum and the amount deposited. 4.6 Calculate the total cost given the percentage sales tax.	4.7 Calculate repeated percentage change e.g., compound interest, depreciation and population increase. 4.8 Reverse percentages e.g., finding the cost price given the selling price and the percentage profit.
5. Personal and House-hold Finance *[Topic Area: Number]*	
All learners should be able to: 5.1 Calculate using money and convert from one currency to another including conversion graphs. 5.2 Use given data to solve problems on simple interest. 5.3 Extract and interpret tables and charts e.g., rates and bills.	5.4 Calculate compound interest (knowledge of compound interest formula is not required).
6. Ratio and Proportions *[Topic Area: Number]*	
All learners should be able to: 6.1 Demonstrate understanding of the elementary ideas and notation of ratio, direct and inverse proportions (variation). 6.2 Divide quantities in a given ratio. 6.3 Use scale in practical situations 6.4 Complete tables for simple direct variation.	6.5 Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities. 6.6 Increase and decrease a quantity by a given ratio.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

7. Indices *[Topic Areas: Number and Algebra]*	
All learners should be able to: 7.1 Use and evaluate positive, negative and zero indices 7.2 Use the rules of indices $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$ and $(a^m)^n = a^{m \times n}$	7.3 Use, interpret and evaluate fractional indices e.g., solve $32^x = 2$.
8. Standard Form *[Topic Area: Number]*	
All learners should be able to: 8.1 Express numbers in standard form $A \times 10^n$ where n is a positive or negative integer and $1 \leq A < 10$. 8.2 Use numbers in standard form.	
9. Properties of Shapes *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 9.1 Use and interpret vocabulary of n-sided: (a) regular polygons, (b) irregular polygons. 9.2 Identify and name prisms and pyramids (cones) using their bases. 9.3 Identify and draw nets of different solids. 9.4 Recognise rotational and line symmetry (including order of rotational symmetry) in 2- dimensions and properties of triangles quadrilaterals and circles related to their symmetries. 9.5 Identify and apply symmetry properties of circles: (a) equal chords are equidistant from the centre, (b) the perpendicular bisector of a chord passes through the centre, (c) tangents from an external point are equal in length. 9.6 Calculate unknown angles using the following geometric properties: (a) angles at a point, (b) angles on a straight line and intersecting lines, (c) angles formed with parallel lines, (d) angle properties of triangles and quadrilaterals, (e) angle between tangent and radius of a circle, (f) angle properties of regular and irregular polygons, (g) angle in a semi-circle, (h) angle at the centre of a circle is twice the angle at the circumference, (i) angles in the same segment are equal, (j) opposite angles in cyclic quadrilaterals are supplementary.	9.7 Use and interpret vocabulary of solid figures: (a) Vertices (b) Edges (c) Faces (d) Net of solid. 9.8 Recognise symmetry properties of the prism and pyramid (including cylinder and cone for each case). 9.9 Identify similar and congruent plane shapes 9.10 Use similarity to calculate a required angle or side. 9.11 Understand and use the relationships between length, area and volume of similar shapes. 9.12 Use geometric properties for simple geometric proofs including congruency and similarity for triangles.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

10. Geometrical terms and Constructions *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 10.1 Understand and use the geometrical terms, point, line, parallel, bearing, right angle, acute, obtuse, reflex, perpendicular, isosceles, equilateral, similarity congruence. 10.2 Understand and use the vocabulary of triangles, quadrilaterals, circles, polygons and simple figures including nets. 10.3 Measure lines and angles. 10.4 Construct a triangle given the three sides using ruler and compasses only. 10.5 Construct other simple geometrical figures from given data using protractors and set squares as necessary. 10.6 Construct angle bisectors and perpendicular bisectors using straight edges and compasses only. 10.7 Read and make scale drawings.	
11. Loci *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 11.1 Construct and describe loci, in 2-dimensions, of points equidistant from (a) a point, (b) two points, (c) a line, (d) two parallel lines, (e) two intersecting lines.	11.2 Describe loci, in 3-dimensions, of points equidistant from: (a) a point, (b) two points, (c) a line, (d) two parallel lines, (e) two intersecting lines. 11.3 Solve problems involving loci.
12. Transformations *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 12.1 Reflect simple plane figures in horizontal or vertical lines. 12.2 Rotate simple plane figures about any point, with given coordinates, through multiples of 90°. 12.3 Construct given translations and enlargements of simple plane figures on a grid. 12.4 Recognise and give precise descriptions of reflections, rotations, enlargements and translations on a grid.	12.5 Reflect simple plane figures in sloping lines. 12.6 Use a stretch and shear on simple figures. 12.7 Carry out combined transformations. 12.8 Use function notation to represent transformations (i.e., reflection (*M*), rotation (*R*), translation (*T*), enlargement (*E*), shear (*H*), stretch (*S*) and their combinations (e.g., *RM*(*A*)). 12.9 Use matrices in transformations (singular matrices are excluded). 12.10 Recognise and give precise descriptions of transformations connecting given figures. 12.11 Describe transformations using coordinates and matrices (singular matrices are excluded).

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

13. Measurement, Time, Units and Mensuration *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 13.1 Use current units of mass, length, area, volume, density and capacity in practical situations and express quantities in terms of larger or smaller units. 13.2 Calculate times in terms of the 24-hour and 12-hour clock , including the conversion between units of time e.g., 2.4 hours = 2 hours 24 minutes. 13.3 Read clocks, dials and timetables. 13.4 Calculate perimeter and area of : (a) rectangles, (b) triangles, (c)circles, (d) parallelogram (e) trapeziums (f) other polygons. (g) composite shapes 13.5 Calculate surface area and volume of: (a) cuboids, (b) cylinders, (c) other prisms. 13.6 Solve problems involving the arc length and sector area as fractions of the circumference and area of a circle.	13.7 Calculate surface area and volume of (a) cones, (b) spheres, (c) Pyramids, (d) composite solids. *Formulae for pyramid, cone, and sphere will be given.* 13.8 Solve problems involving solids (e.g., hollow solids and truncated solids-frustums).
14. Trigonometry *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 14.1 Apply Pythagoras Theorem. 14.2 Calculate sides and angles of a right-angled triangle using sine, cosine and tangent ratios. 14.3 Solve simple problems involving angles of depression and elevation (from right angled-triangles).	14.4 Find sine and cosine for obtuse angles. 14.5 Use sine rule and the cosine formulae for trigonometrical problems in 2-dimensions. 14.6 Use the formulae $A = \frac{1}{2} ab \sin C$ for the area of a triangle. 14.7 Solve trigonometric problems in 3-dimensions including angle between a line and a plane.
15. Bearings *[Topic Area: Shape, Position and Space]*	
All learners should be able to: 15.1 Draw and interpret bearings. 15.2 Interpret and use three-figure bearings measured clockwise from the North (i.e., 000° to 360°). 15.3 Apply bearings to solve simple problems involving not more than three north lines. 15.4 Find distances and make simple scale drawings.	

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

16. Graphs in Practical situations *[Topic Areas: Algebra and Shape, Position and Space]*

All learners should be able to:

16.1 Draw and use simple graphs in practical situations including conversion graphs and distance time graphs.
16.2 Calculate speed from a distance-time graph.
16.3 Construct/draw a speed-time graph.
16.4 Interpret speed-time graphs including acceleration and area under the graph being distance.

17. Vectors *[Topic Areas: Algebra and Shape, Position and Space]*

All learners should be able to:

17.1 Understand representation of vectors by a directed line segment.
7.2 Represent a vector by $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\overrightarrow{AB}$ or $\underline{a}$.
17.3 Add and subtract vectors.
17.4 Multiply a vector by a scalar.
17.5 Calculate the magnitude/ length of a vector and use the notation $|\underline{a}|$ to represent vector magnitude or length of vector.
17.6 Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors.
17.7 Identify parallel vectors as those that are scalar multiples of each other.
17.8 Simplify vector expressions.
17.9 Use base vectors to represent any vector on the plane.
17.10 Use position vectors.
17.11 Identify collinear points using vectors.

18. Algebraic representation and formulae *[Topic Area: Algebra]*

All learners should be able to:

18.1 Use letters for numbers to express generalised numbers and expressions algebraically.
18.2 Substitute numbers for words and letters in formulae.
18.3 Change the subject of simple formulae.
18.4 Construct simple algebraic expressions and set up simple equations.
18.5 Expand brackets including double brackets
18.6 Simplify algebraic expressions
e.g., $5(x + 3) - 2(x - 5)$.
18.7 Factorise two-term expressions using a common factor.
18.8 Factorise expressions of the form $x^2 + bx + c$.
18.9 Simplify algebraic fractions of the form
$\frac{ax}{k_1} \pm \frac{bx}{k_2}$ or $\frac{x \pm a}{k_1} \pm \frac{x \pm b}{k_2}$, $k_1, k_2 \neq 0$
e.g., $\frac{2x}{3} - \frac{x}{4}$, $\frac{2}{3a} + \frac{4}{5a}$;
or of the form e.g., $\frac{2}{3a} \times \frac{4}{5a}$, $\frac{2x}{3} \div \frac{x}{4}$.
18.10 Construct and simplify more complicated formulae and equations (square and cubic).
18.11 Change the subject of a formula;
(i) when the new subject appears on both sides by collecting like terms and factorizing.
(ii) when there is a square root or a square or a cube.
18.12 Simplify algebraic fractions of the form
$\frac{k}{x - a} \pm \frac{w}{x + b}$
(two term denominator)
18.13 Factorise expressions of the form
$ax + bx + kay + kby$, $a^2x^2 - b^2y^2$,
$p^2 + 2pq + q^2$ and $ax^2 + bx + c$, where $a \neq 1$
(i.e., a can be negative or greater than 1).
18.14 Simplify algebraic fractions where numerator and denominator are quadratic expressions such as
$\frac{hx^2 - kx}{ax^2 - bx + c}$, e.g., $\frac{x^2 - 2x}{x^2 - 5x + 6}$.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations.

19. Coordinates, graphs, relations and function notation *[Topic Area: Algebra]*

All learners should be able to:

19.1 Use the words domain and range interchangeably with input and output respectively.
19.2 Use basic linear language for linear functions e.g., $f(x) = x + 3$ or $f: x \mapsto x + 3$.
19.3 Evaluate a linear function by substituting for a given value of the domain.
19.4 Describe the inverse of a linear function using a flow chart.
19.5 Find the inverse of a linear function algebraically.
19.6 Appreciate equivalences of $f(x) = ...$ and $y = ...$
19.7 Construct tables of values and draw and recognise graphs for functions of the form $ax + b$, $\pm x^2 + ax + b$, and $\frac{a}{x}$ $(x \neq 0)$ where a and b are integral constants.
19.8 Calculate the gradient of a straight line.
19.9 Estimate gradients of curves by drawing tangents.
19.10 Interpret and obtain the equation of a straight line in the form $y = mx + c$.
19.11 Identify the gradient and the y-intercept when an equation of line is given in the form $y = mx + c$.
19.12 Form a composite function of the form $fg(x)$ given g and f by first applying function g on x, and then function f on $g(x)$.
19.13 Evaluate composite functions by substitution.
19.14 Find the inverse composite functions using:
(i) the flow diagram,
(ii) the algebraic method.
19.15 Relate a quadratic mapping to a function using $y = ...$ and $f(x) = ...$
19.16 Calculate the length of a straight line segment from the coordinates of its end points.
19.17 Calculate the coordinates of the midpoint of a line segment given its end points.
19.18 Construct tables of values and draw and interpret graphs for the function of the form $y = ax^n$ and $y = ax^n + bx + c$, where a is a rational constant and $n = -2, -1, 0, 1, 2, 3$ (a,b,c are constants where a is not equal to zero).
19.19 Construct tables and draw and interpret functions of the form a^x where a is a positive integer.
19.20 Solve equations associated with curve graphs approximately by graphical methods.

20. Solution of equations and inequalities *[Topic Area: Algebra]*

All learners should be able to:

20.1 Form and solve simple linear equations e.g., $3x + 2 = 14$, $5(2x - 3) - 3(x + 4) = 2$.
20.2 Solve linear fractional equations with numerical denominators.
20.3 Solve simple linear inequalities including fractional inequalities with numerical denominators.
20.4 Form and solve simultaneous linear equation in two unknowns by:
(i) the method of substitution,
(ii) the method of elimination/addition,
(iii) graphical method.
20.5 Solve linear equations by graphical methods.
20.6 Solve quadratic equation of the form $x^2 + bx + c = 0$ by:
(i) the method of factorisation,
(ii) graphical method.
20.7 Form and solve non-linear equations (e.g., quadratic) from statements by first defining an unknown.
20.8 Solve double linear inequalities e.g., $-5 < 2x + 3 \leq 10$.
20.9 Solve simultaneous inequalities e.g., $2x + 3 < 10$ and $x + 7 \geq 2$.
20.10 Solve quadratic equations of the form $ax^2 + bx + c = 0$ where $a \neq 1$ by:
(i) graphical method,
(ii) factorisation,
(iii) completing the square,
(iv) the quadratic formula.
20.11 Solution of simultaneous linear equations by matrix method.
20.12 Solve non-linear fractional equations with linear algebraic denominators.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

21. Matrices *[Topic Areas: Algebra, Data Handling and Shape, Position and Space]*	
All learners should be able to: 21.1 Display information in the form of a matrix of any order. 21.2 Calculate the product of a matrix and a scalar. 21.3 Perform basic operations: addition, subtraction and multiplication (except division) on matrices of any order (where compatible). 21.4 Understand and use the zero and identity 2 x 2 matrices. 21.5 Use equality of matrices in simple matrix equations.	21.6 Use the algebra of 2 x 2 matrices including the zero and identity matrices. 21.7 Calculate and use the determinant and inverse of a 2 × 2 matrix (non-singular matrix). 21.8 Solve simultaneous equations using matrices. 21.9 Use matrices in transformations.
22. Linear programming *[Topic Areas: Algebra, Data Handling and Shape, Position and Space]*	
All learners should be able to: 22.1 Represent graphically single linear inequalities in one or two variables. *The convention of using broken lines for < and > and solid lines for ≤ and ≥ will be expected.* 22.2 Form inequalities from graphs of single regions by first determining the equation of the boundary line.	22.3 Represent graphically the solution set to 2 or more simultaneous inequalities in one or two variables. *The convention of using broken lines for < and > and solid lines for ≤ and ≥ will be expected.* 22.4 Form inequalities from graphs of regions by first determining the equations of the boundary lines. 22.5 Solve simple linear programming problems by representing the information in inequality form and drawing graphs of these inequalities.
23. Statistics *[Topic Area: Data Handling]*	
All learners should be able to: 23.1 Collect classify and tabulate data. 23.2 Read, interpret and draw simple inferences from tables and diagrams. 23.3 Find the mean and median from ungrouped data. 23.4 Construct and use bar charts for qualitative and numerical data (discrete and grouped with equal intervals). 23.5 Construct and use pie charts. 23.6 Construct scatter diagrams (including drawing a line of best fit). 23.7 Understand what is meant by positive, negative and zero correlation.	23.11 Distinguish the purpose for which mean, median and mean are used. 23.12 Draw frequency polygons from frequency tables. 23.13 Construct and use histograms (equal and unequal intervals). *Frequency Density is expected on the vertical axis.* 23.14 Construct a cumulative table for ungrouped and grouped data. 23.15 Draw cumulative frequency diagrams.

23.8 Calculate the mean, median and mode for discrete data. 23.9 Calculate the range. 23.10 Construct frequency table for grouped data (a) for qualitative data (e.g., who liked what?), (b) quantitive data, (discrete and grouped data with equal intervals).	23.16 Find median, quartiles percentiles and inter-quartile range from cumulative frequency diagrams and distinguish the purpose for which they are used. 23.17 Find median, quartiles and interquartile range for discrete data from a cumulative frequency table. 23.18 Find the mean, and modal class for grouped data (from frequency tables).
24. Probability ***[Topic Area: Data Handling]***	
All learners should be able to: 24.1 Explain the terms and phrases used in probability. 24.2 Calculate the probability of a single event as either a fraction or a decimal (not a ratio). 24.3 Understand and use probability scale from 0 to 1. 24.4 Understand that the probability of *an event occurring is 1* – (minus) *the probability of an event not occurring.* 24.5 Understand that relative frequency approximates to probability provided the number of trials is large enough. 24.6 Use the basic rules of probability for the combined events *A and B* and *A or B.* 24.7 Find probabilities of two combined events using possibility space diagrams (outcomes represented by points on a grid).	24.8 Find probabilities of simple combined events using tree diagrams (independent and dependent events). 24.9 Use relative frequency as probability in practice (e.g., frequency and, cumulative frequency tables).

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

GRADE DESCRIPTIONS

The scheme of assessment is intended to encourage positive achievement by all candidates. Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The grade awarded will depend on the extent to which the candidate has met the assessment objectives overall.

Criteria for the standard of achievement likely to have been shown by candidates awarded Grades A, C and F are shown below.

A Grade A candidate should be able to:

- Use positive, negative and fractional indices in both numerical and algebraic work. Interpret and use fractional indices, both numerical and algebraic.
- Express any number to 1, 2, or 3 significant figures.
- Relate a percentage change to a multiplying factor and vice versa, e.g., multiplication by 1.03 results in a 3% increase.
- Obtain appropriate upper and lower bounds for solutions to simple problems given data to a specified accuracy.
- Solve problems involving solids including nets of solids.
- Relate scale factors to situations in both two and three dimensions.
- Calculate actual length, areas and volumes from scale models. Carry out calculations involving the use of right-angled triangles as part of work in three dimensions.
- Add, subtract, multiply and divide algebraic fractions.
- Manipulate algebraic equations – linear, simultaneous and quadratic.
- Write down algebraic formulae and equations from a description of a situation.
- Form and solve double linear inequalities.
- Recognise and interpret graphs of the functions $f(x) = ax^n$, and $g(x) = a^x$
- Plot, recognise and interpret graphs of functions of the form $f(x) = ax^n + bx + c$
- Draw the tangent, estimate and interpret gradients of a curve at a point.
- Solve problems involving the sine formula, cosine formula and the use of the area formula for a triangle $A = \frac{1}{2} ab \sin C$
- Use the relationships between lengths of line segments, areas, surface area, volume of similar shapes or solids to solve problems.
- Construct and interpret histograms with unequal intervals.
- Calculate the probability of simple combined events, using addition or multiplication of probabilities as appropriate.
- Recognise, describe and generalise in algebraic format patterns of non-linear sequences.
- Process data, discriminating between necessary and redundant information. Make quantitative and qualitative deductions distance/time and speed/time graphs.
- Make clear, concise and accurate mathematical statements, demonstrating ease and confidence in the use of symbolic forms and accuracy in algebraic or arithmetic manipulation.
- Give clear mathematical justifications for the conjectures made in problem solving.

Appendix 1

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

A Grade C candidate should be able to:

- Apply the four rules of number to positive and negative integers, and vulgar and decimal fractions.
- Use positive and negative indices in numerical work.
- Calculate percentage change.
- Perform calculations involving several operations.
- Use a calculator fluently. Give a reasonable approximation to a calculation involving the four rules.
- Use and understand the standard form of a number.
- Use area and volume units.
- Find the volume and surface area of a prism and a cylinder.
- Use a scale diagram to solve a two-dimensional problem.
- Solve ratio and proportion problems.
- Solve problems involving perimeters and areas of compound shapes bounded by line segments and/or circular arcs.
- Solve practical problems involving mass, volume and density.
- Draw distance-time graphs.
- Make quantitative and qualitative conclusion from distance-time graphs.
- Manipulate algebraic fractions with denominators containing a single term (numerical or algebraic).
- Form simple algebraic expressions
- Factorise two-term expressions and expressions of the form $x^2 + bx + c$.
- Form and solve linear equations in practical situations.
- Manipulate and solve fractional equations and quadratic equations (a =1).
- Calculate the length of the third side of a right-angled triangle.
- Find the angle in a right-angled triangle, given two sides.
- Calculate angles in geometrical figures.
- Recognise, and in simple cases formulate, rules for generating a pattern or sequence.
- Solve simple simultaneous linear equations in two unknowns.
- Form and solve simple linear inequalities
- Represent regions in the plane determined by linear inequalities.
- Identify and describe rotational symmetry in two dimensions.
- Use angle properties and symmetry properties of a circle to calculate specified angles and/or length of line segments.
- Use cosine, sine and tangent ratios in right angled triangles when solving problems in two dimensions (including bearings, angles of elevation and depression).
- Draw and or state loci of points in two dimensions.
- Draw, recognise and describe transformations of shapes (translation, rotation, reflection and enlargement).
- Make, use and interpret scale drawings.
- Find the magnitude of a vector.
- Calculate the probability of single events.
- Make and justify estimates of probability.
- Understand that relative frequency approximates to probability.
- Analyse a given situation, generate data, generalise the data and describe the situation using mathematical symbols, words or diagrams.
- Transform simple formulae.
- Substitute numbers in more difficult formulae and evaluate the remaining term.
- Use brackets and extract common factors from algebraic expressions.
- Construct a pie-chart from simple data.
- Plot and interpret graphs, including travel graphs, conversion graphs and graphs of linear and simple quadratic functions.

SGCSE MATHEMATICS Syllabus 6880
November 2009 and November 2010 Examinations

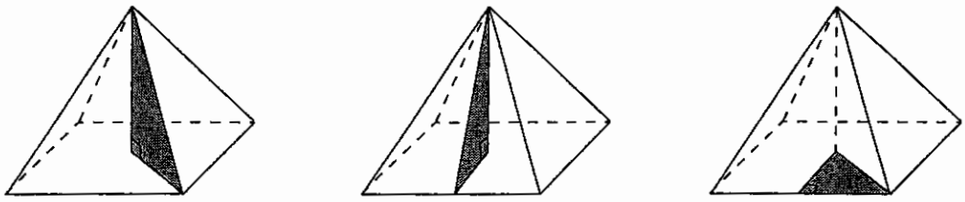
A Grade F candidate should be able to:

- Perform the four rules on positive integers and decimal fractions (one operation only) using a calculator where necessary.
- Convert a fraction to a decimal.
- Calculate a simple percentage.
- Use metric units of length, mass and capacity.
- Understand the relationship between mm, cm, m, km, g, and kg.
- Continue a straightforward number sequence.
- Find the perimeter and area of a rectangle and other rectilinear shapes.
- Draw a triangle given three sides.
- Measure a given angle.
- Substitute numbers in a simple formula and evaluate the remaining terms.
- Solve simple linear equations in one unknown.
- Extract information from simple timetables.
- Tabulate numerical data to find the frequency of given scores.
- Draw a bar chart.
- Plot given points.
- Read travel graph.
- Calculate the mean of a set of numbers.
- Manipulate simple algebraic expressions.
- Recognise and name plane shapes ad solids.
- Recognise, describe and reflect plane shapes on a coordinate grid in lines parallel to the axes.
- Translate plane shapes on a plane grid.
- Identify and use reflectional symmetry in two dimensional shapes.
- Represent, add and subtract vectors. Multiply column vectors by a scalar.
- Read tables, graphs (including travel graphs) and diagrams.

4.9a Calculating distances and angles in solids

Finding distances and angles in solids involves solving right-angled triangles viewed in different planes, making use of Pythagoras' theorem and the trigonometrical ratios sin, cos and tan.

It is very important to draw a simple diagram that helps you to see clearly the part of the solid that you are working on and the type of operation you need to perform. Your drawing should include all the information you need to answer the question and you should clearly label the things you need to find. Ignore what you don't need. Draw vertical lines up and down the page, and if one point is vertically below another then draw it that way. Make all parallel lines parallel.

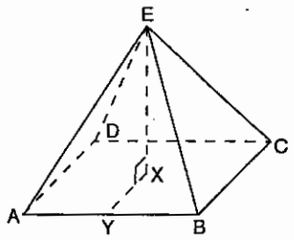


Remember 'A good picture is worth a thousand words.'

In sketches of three-dimensional solids right angles are not always obvious. There are often words in the question which will help you to identify the right angles: one line may be *perpendicular* to another, a point may be *vertically below* or *vertically above* another point.

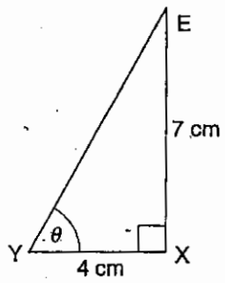
E.g. In a regular square based pyramid ABCDE the edge AB = 8 cm and the vertical height to the apex point E = 7 cm. Let X be the point on the base directly below E and let Y be the mid-point of AB.

(a) Calculate the angle EYX.
(b) Calculate the length of the slope EY.
(c) Calculate the area of the face ABE.



(a) To calculate ∠ EYX you need to draw the triangle EXY.

You know that EX = 7 cm. Because EX is vertical the angle EXY is a right angle. Because the pyramid is regular, X is in the middle of the base so XY is half the length of the base. Therefore XY = 4 cm.



Looking at the diagram you can see that you know the opposite and adjacent sides to the angle you want to find. So use tan.

Let ∠ EYX = θ

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{4}$$

7 ÷ 4 = INV tan

∠ EYX = 60·26° (to 2 d.p.)

(b) To calculate the length of the slope EY you can use Pythagoras' theorem in the triangle already drawn. (You could use the angle you have just found to calculate EY but if you got the angle wrong then you'd get the length of EY wrong too.)

Using Pythagoras, $EY^2 = EX^2 + XY^2$

$= 4^2 + 7^2$

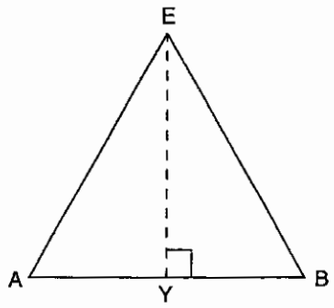
$= 16 + 49$

$= 65$

$EY = \sqrt{65}$

EY = 8·06 cm (to 2 d.p.)

(c) To find the area of the triangle ABE you need a new diagram.



You know the length of EY from part (b).

The exact value of EY is $\sqrt{65}$ cm. You know that AB = 8 cm.

Area of ABE = $\frac{1}{2}$ base × height

$= \frac{1}{2} \times 8 \times \sqrt{65}$

$= 32{\cdot}249031$

The area of ABE = 32·25 cm² (to 2 d.p.)

?

1 ABCDEFGH is a regular cuboid with AB = 25 cm, BC = 10 cm and BF = 7 cm.

(a) Find the length EG.

Hint: Draw triangle EFG.

(b) Find the length AG.

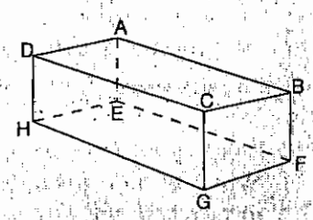
Hint: Draw triangle AEG.

(c) Calculate the angle AGE.

(d) Find the length BG.

(e) Calculate the angle AGB.

(f) Calculate the area of the triangle ABG.



2 A snail is moving on a brick represented by the cuboid in question 1. The snail travels in a straight line from A to G along the shortest route.

(a) Calculate the length of the snail's trail.

Hint: Imagine the box opened out.

(b) Find the angle from AB that the snail will travel along.

(c) How far from the corner D will the snail cross the edge CD?

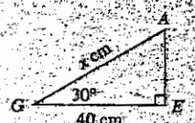
Three-dimensional problems

Example

Consider the following diagram, in which we want to find

When finding side lengths and angles in three-dimensional problems, follow these steps:

Step 1 Draw the relevant triangle from the main diagram.



Step 2 Write down the appropriate ratio.

$$\cos 30° = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Step 3 Put in the appropriate details and solve.

$$\cos 30° = \frac{40}{x}$$

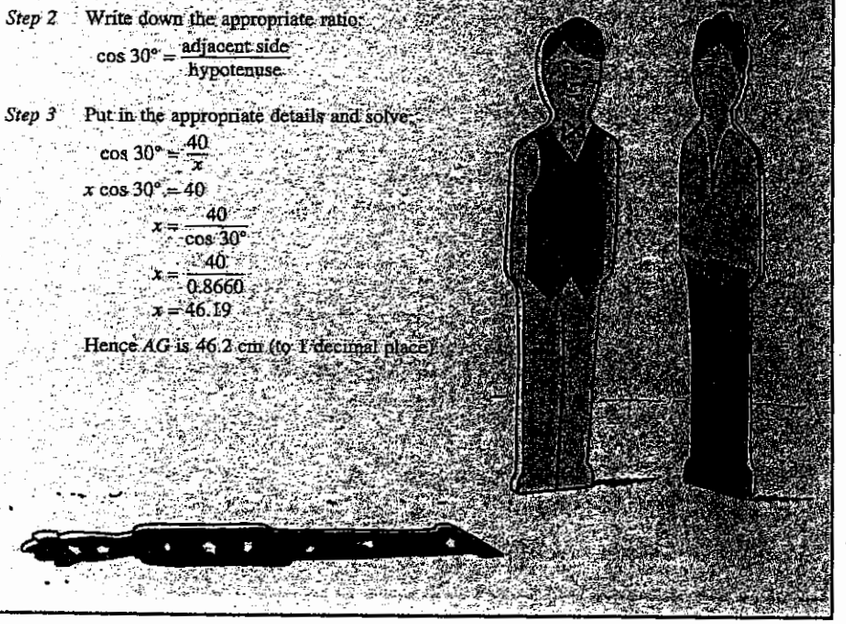
$$x \cos 30° = 40$$

$$x = \frac{40}{\cos 30°}$$

$$x = \frac{40}{0.8660}$$

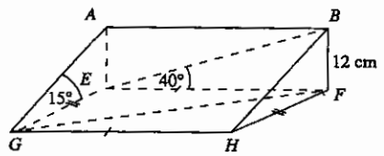
$$x = 46.19$$

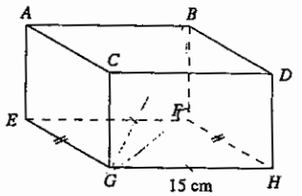
Hence AG is 46.2 cm (to 1 decimal place).



Skills practice

For each of the following diagrams find the distances and angles listed.

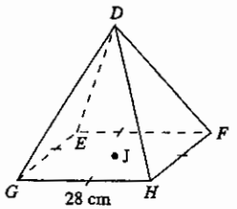




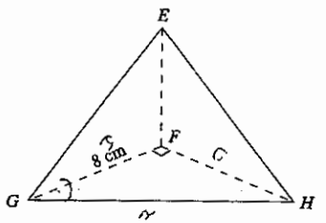
$\angle DGH = 30°$ and $\angle FGH = 40°$

1 AE **2** EG
3 EF **4** GF

5 GD **6** FG
7 DH **8** GB



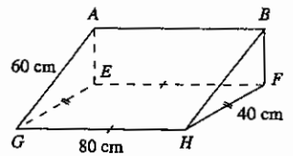
$\angle DGH = 60°$ and $\angle FGH = 45°$.

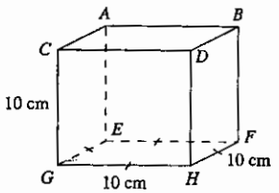


$\angle FGH = 30°$ and $\angle EGF = 42°$.

9 FH **10** DG **11** DJ

12 FH **13** GH
14 EG **15** EF



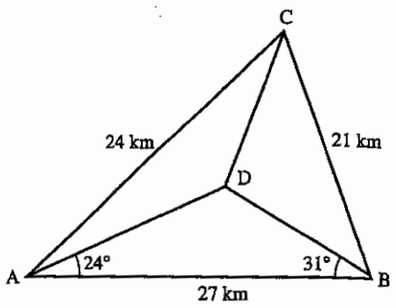


16 $\angle AGE$ **17** $\angle BEF$
18 $\angle BGF$

19 $\angle AGE$ **20** $\angle BGF$
21 $\angle BEF$

5. A control tower observes the flight of an unidentified flying object.
At 09:23 the U.F.O. is 580 km away on a bearing of 043°.
At 09:25 the U.F.O. is 360 km away on a bearing of 016°.
What is the speed and the course of the U.F.O.?
[Use a scale of 1 cm to 50 km]

6. Make a scale drawing of the diagram below and find the length of CD in km.

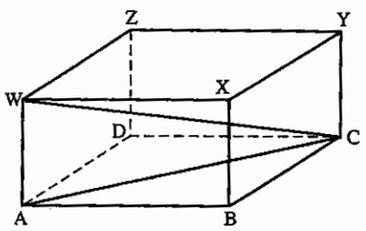


6.3 Three-dimensional problems

Always draw a large, clear diagram. It is often helpful to redraw the triangle which contains the length or angle to be found.

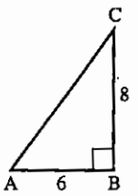
Example

A rectangular box with top WXYZ and base ABCD has AB = 6 cm, BC = 8 cm and WA = 3 cm.
Calculate:
(a) the length of AC
(b) the angle between WC and AC.



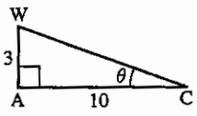
(a) Redraw triangle ABC.

$AC^2 = 6^2 + 8^2 = 100$
$AC = 10$ cm



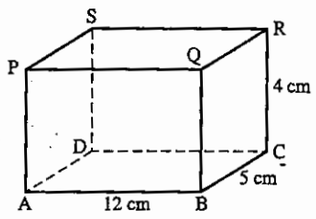
(b) Redraw triangle WAC.

Let $W\hat{C}A = \theta$
$\tan\theta = \frac{3}{10}$
$\theta = 16{\cdot}7°$
The angle between WC and AC is 16·7°.

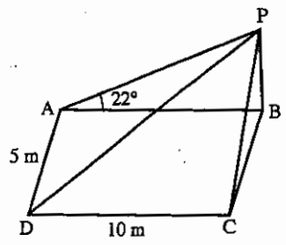


Exercise 7

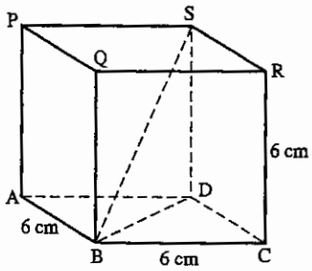
1. In the rectangular box shown, find:
(a) AC
(b) AR
(c) the angle between AC and AR.



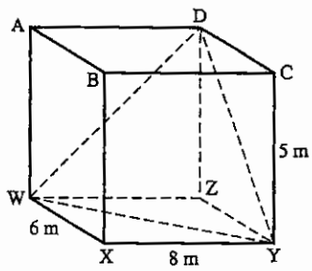
2. A vertical pole BP stands at one corner of a horizontal rectangular field as shown.
If AB = 10 m, AD = 5 m and the angle of elevation of P from A is 22°, calculate:
(a) the height of the pole
(b) the angle of elevation of P from C
(c) the length of a diagonal of the rectangle ABCD
(d) the angle of elevation of P from D.



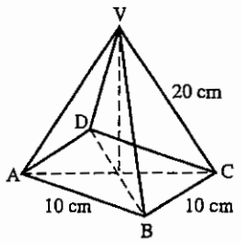
3. In the cube shown, find:
(a) BD
(b) AS
(c) BS
(d) the angle SBD
(e) the angle ASB



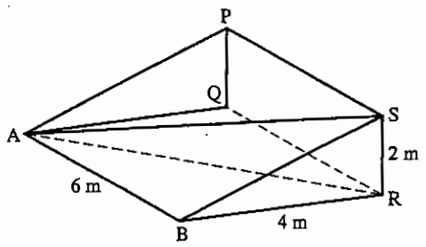
4. In the cuboid shown, find:
(a) WY
(b) DY
(c) WD
(d) the angle WDY



5. In the square-based pyramid, V is vertically above the middle of the base, AB = 10 cm and VC = 20 cm. Find:
(a) AC
(b) the height of the pyramid
(c) the angle between VC and the base ABCD
(d) the angle AVB
(e) the angle AVC



6. In the wedge shown, PQRS is perpendicular to ABRQ; PQRS and ABRQ are rectangles with AB = QR = 6 m, BR = 4 m, RS = 2 m. Find:
(a) BS (b) AS
(c) angle BSR (d) angle ASR
(e) angle PAS



7. The edges of a box are 4 cm, 6 cm and 8 cm. Find the length of a diagonal and the angle it makes with the diagonal on the largest face.