UNIVERSITY OF SWAZILAND
Faculty of Health Sciences

DIPLOMA/ BSc ENVIRONMENTAL HEALTH SCIENCES
FINAL EXAMINATION PAPER 2005

TITLE OF PAPER   :   HEALTH SCIENCES MATHEMATICS HEALTH

COURSE CODE   :   HSM 110

DURATION   :   3 HOURS

MARKS   :   100

INSTRUCTIONS   :   READ THE QUESTIONS & INSTRUCTIONS CAREFULLY

      :   ANSWER ANY **FIVE** QUESTIONS

      :   EACH QUESTION CARRIES 20 MARKS.

      :   A SET OF FORMULAE WILL BE PROVIDED

      :   WRITE NEATLY & CLEARLY

      :   NO PAPER SHOULD BE BROUGHT INTO OR OUT OF THE EXAMINATION ROOM.

      :   BEGIN EACH QUESTION ON A SEPARATE SHEET OF PAPER.

DO NOT OPEN THIS QUESTION PAPER UNTIL PERMISSION IS GRANTED BY THE INVIGILATOR.

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Question 1

(a) Find the quotient $Q(x)$ and remainder $R(x)$ when $P(x) = x^5 - x^4 + x^3 - x + 1$ is divided by $D(x) = x^2 + 1$.

[10 marks]

(b) Evaluate the following limits:

(i) $\displaystyle \lim_{x \to 3} \frac{x^2 - 4}{x - 2}$

(ii) $\displaystyle \lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

[10 marks]

Question 2

(a) Find all values of k such that $x - 1$ is a factor of $4kx^3 + k^2 x^2 - x + 4$.

[10 marks]

(b) Find the middle term in the expansion of $\left(x - \dfrac{2x}{y} \right)^{16}$.

[10 marks]

Question 3

(a) Prove the trigonometric identity: $\dfrac{2 \tan x}{1 + \tan^2 x} = 2 \sin x \cos x$.

[10 marks]

(b) Find all roots of the polynomial equation $4x^3 - 16x^2 + 19x - 6$.

[10 marks]

Question 4

(a) Convert to a product: $\cos 20^o - \cos 40^o$.

[8 marks]

(b) For the function , find and classify all stationary points, determine intervals of increase and decrease, and sketch its graph: $y = x^3 - 2x^2 + x$.

[12 marks]

Question 5

(a) Let A be an angle in Q-III (third quadrant). Give exact values of $\sin A$ and $\cos 2A$, given that $\cos A = -\dfrac{5}{13}$.

[10 marks]

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Q5 (b) Find all roots of the polynomial equation: $2x^3 - 3x^2 - 3x + 2$.

[10 marks]

Question 6

(a) Differentiate the following: $y = \dfrac{x^3 - 1}{x^3 + 1}$.

[5 marks]

(b) Use EITHER Gaussian Elimination OR Crammer's rule to solve the following linear system of equations:

$$\begin{cases} x_1 + 2x_2 + 3x_3 = \quad 2 \\ 2x_1 + 5x_2 + 7x_3 = \quad 1 \\ -2x_1 - 4x_2 - 5x_3 = -1 \end{cases}$$

[10 marks]

(c) Find value(s) of c such that the vectors $a = (c, 3c, 1)$ and $b = (2, -c, 5)$ are orthogonal.

[5 marks]

Question 7

(a) Use the definition of the derivative to find $f'(x)$, given that $f(x) = x^2 - 1$.

[10 marks]

(b) A ladder 10 m long leans against a vertical wall. If the bottom of the ladder is being pulled away from the wall at a rate of 50 cm/s, how quickly is the top of the ladder descending down the wall when the base of the ladder is 4 m from the wall?

[10 marks]

Question 8

(a) For the following function, evaluate y'':

$$y = \sqrt{x^2 - 1}.$$

[5 marks]

(b) Differentiate the following:

$$y = e^x \ln(x^2 - x).$$

[5 marks]

(c) A farmer has 600 m of fencing available. He intends to use it to fence off three sides of a rectangular field in which the fourth side is bounded by a stream and is not to be fenced. Find the field of maximum area that can be fenced with the fencing available.

[10 marks]

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