

**UNIVERSITY OF SWAZILAND**



**Final Examination 2006**

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**Title of Paper** : Health Sciences Mathematics

**Program** : Dip. Env.Health I/ Dip. Env.Health IV

**Course Number** : HSM 110

**Time Allowed** : Three (3) Hours

**Instructions** :

1. This paper consists of EIGHT (8) questions on TWO (2) pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

**Special Requirements** : None

**THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.**

**Question 1**

(a) Find all values of  $k$  such that  $x - 1$  is a factor of  $4kx^3 + k^2x^2 - x + 4$ .

[10 marks]

(b) Differentiate the following:

(i)  $y = \frac{x^3 - 1}{x^3 + 1}$

(ii)  $y = \sqrt{x^3 - 5x^2 + x}$

[10 marks]

**Question 2**

(a) Find all roots of the polynomial equation  $x^3 - 2x^2 - x + 2 = 0$ .

[10 marks]

(b) Use EITHER Gaussian Elimination OR Cramer's rule to solve the following linear system of equations:

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ 2x_1 + 5x_2 + 7x_3 = 1 \\ -2x_1 - 4x_2 - 5x_3 = -1 \end{cases}$$

[10 marks]

**Question 3**

(a) Use the definition of the derivative to find  $f'(x)$ , given that  $f(x) = x^2 - 2x$ .

[10 marks]

(b) Find the equation of the line tangent to  $y = x^2 - \sqrt{x}$  at  $(4, 14)$ .

[5 marks]

(c) For the following function, evaluate  $y''$ :

$$y = \sqrt{2x+1}$$

[5 marks]

**Question 4**

(a) Convert the difference to a product:  $\sin 55^\circ - \sin 35^\circ$ .

[8 marks]

(b) For the function, find and classify all stationary points, determine intervals of increase and decrease, and sketch its graph:  $y = x^4 - 4x^3 + 1$ .

[12 marks]

**Question 5**

- (a) Let  $A$  be an angle in  $Q-III$  (third quadrant). Give exact values of  $\sin A$  and  $\cos 2A$ , given that

$$\cos A = -\frac{12}{13}.$$

[10 marks]

- (b) Sketch the region enclosed by the curves, and find its area:  $y = 2 - x^2$ ,  $y = -x$ .

[10 marks]

**Question 6**

- (a) Expand and simplify:  $\left(\frac{1}{x} - 2x\right)^4$ .

[8 marks]

- (b) Find the equation of the straight line through  $(-2, -4)$  which is parallel to the line  $8x - 2y + 3 = 0$ .

[7 marks]

- (c) Find value(s) of  $c$  such that the vectors  $a = (c, 2c, 1)$  and  $b = (1, c, -1)$  are orthogonal.

[5 marks]

**Question 7**

- (a) Determine the centre and radius of the circle:  $4x^2 + 4y^2 + 80x + 12y + 265 = 0$ .

[10 marks]

- (b) A rectangular plot is to be fenced using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for E3 per metre, while the remaining sides will use standard fencing selling for E2 per metre. What are the dimensions of the plot of greatest area that can be fenced for E6000?

[10 marks]

**Question 8**

- (a) A copper ball is heated to a temperature of  $100^\circ\text{C}$ . Then, at time  $t = 0$  it is placed in water that is maintained at a temperature of  $30^\circ\text{C}$ . At the end of 3 minutes, the temperature of the ball has been reduced to  $70^\circ\text{C}$ . Find the time at which the temperature of the ball is reduced to  $31^\circ\text{C}$ .

[10 marks]

- (b) Evaluate the following indefinite integrals:

(i)  $\int (2 - x^4 + \sqrt{x}) dx$

(ii)  $\int x\sqrt{4 - 3x^2} dx$ .

[10 marks]