# UNIVERSITY OF SWAZILAND

## Supplementary Examination 2006

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| **Title of Paper** | : | Health Sciences Mathematics |
| **Program** | : | Dip. Env.Health I/ Dip. Env.Health IV |
| **Course Number** | : | HSM 110 |
| **Time Allowed** | : | Three (3) Hours |
| **Instructions** | : | |

1. This paper consists of EIGHT (8) questions on TWO (2) pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

**Special Requirements** : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION
TO DO SO IS GRANTED BY THE INVIGILATOR.

## Question 1

(a) Find the quotient $Q(x)$ and remainder $R(x)$ when $P(x) = x^6 - x^4 + x^3 - x + 1$ is divided by $D(x) = x^2 + 1$.

[10 marks]

(b) Evaluate the following limits:

(i) $\lim\limits_{x \to 3} \dfrac{2x^2 - 5x - 7}{x^2 - 3x + 2}$

(ii) $\lim\limits_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

[10 marks]

## Question 2

(a) Let $A$ be an angle in $Q$-$IV$ (fourth quadrant). Give exact values of $\tan A$ and $\cos 2A$, given that $\sin A = -\dfrac{3}{5}$.

[10 marks]

(b) Find the middle term in the expansion of $\left( y - \dfrac{x}{y^2} \right)^{12}$.

[10 marks]

## Question 3

(a) Prove the trigonometric identity: $\dfrac{2 \tan x}{1 + \tan^2 x} = 2 \sin x \cos x$.

[10 marks]

(b) Find all roots of the polynomial equation $x^3 - 2x^2 - x + 2 = 0$.

[10 marks]

## Question 4

(a) Convert to a sum/difference: $\sin 80^o \sin 20^o$.

[8 marks]

(b) For the function , find and classify all stationary points, determine intervals of increase and decrease, and sketch its graph: $y = x^3 - 3x + 1$.

[12 marks]

## Question 5

(a) Find the equation of the line tangent to $y = x^2 - \sqrt{x}$ at (4, 14).

[10 marks]

Q5 (b)  Use the binomial series to evaluate, correct to 4 decimal places: $\sqrt[4]{82}$ .

[10 marks]

**Question 6**

(a)  Differentiate the following:  $y = \dfrac{x^2 + 1}{x^2 - 1}$ .

[5 marks]

(b)  Use EITHER Gaussian Elimination OR Crammer's rule to solve the following linear system of equations:

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 7 \\ x_1 + 5x_2 + 7x_3 = 20 \\ -3x_1 + x_2 - 5x_3 = -12 \end{cases}$$

[10 marks]

(c)  Find value(s) of $c$ such that the vectors $a = (c, -2, 1)$ and $b = (2c, c, -4)$ are orthogonal.

[5 marks]

## Question 7

(a)  Use the definition of the derivative to find $f'(x)$, given that $f(x) = x^2 - x$.

[10 marks]

(b)  A ladder 10 m long leans against a vertical wall.  If the bottom of the ladder is being pulled away from the wall at a rate of 50 cm/s, how quickly is the top of the ladder descending down the wall when the base of the ladder is 4 m from the wall?

[10 marks]

**Question 8**

(a) For the following function, evaluate $y''$ :

$$y = \sqrt{2x - 1} .$$

[5 marks]

(b) Differentiate the following:

$$y = e^{2x} \ln(x^2 - 2x) .$$

[5 marks]

(c)  A new property owner has 800 m of fencing with which to fence off his rectangular plot of land.  If the shape of the plot is rectangular, and three sides are to be fenced (as neighbours already have a fence bordering the third side), determine the dimensions of the field of maximum area that can be enclosed by the fencing available.

[10 marks]

# Useful Formulae

1. $\sin^2 x + \cos^2 x = 1$

2. $1 + \tan^2 x = \sec^2 x$

3. $1 + \cot^2 x = \csc^2 x$

4. $\cos 2x = \cos^2 x - \sin^2 x$

5. $\cos 2x = 1 - 2\sin^2 x$

6. $\cos 2x = 2\cos^2 x - 1$

7. $\sin 2x = 2 \sin x \cos x$

8. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

9. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

10. $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

11. $\cos(x - y) + \cos(x + y) = 2 \cos x \cos y$

12. $\cos(x - y) - \cos(x + y) = 2 \sin x \sin y$

## Special Angles

| Angle | sin | cos | tan |
|-------|-----|-----|-----|
| $0^o$ | 0 | 1 | 0 |
| $30^o$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ |
| $60^o$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| $90^o$ | 1 | 0 | Undefined |