

UNIVERSITY OF SWAZILAND



Final Examination 2007

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- Title of Paper** : Health Sciences Mathematics
- Program** : Dip. Env.Health I/ Dip. Env.Health IV
- Course Number** : HSM 110
- Time Allowed** : Three (3) Hours
- Instructions** :
1. This paper consists of EIGHT (8) questions on TWO (2) pages.
 2. Answer any five (5) questions.
 3. Non-programmable calculators may be used.
- Special Requirements** : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

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QUESTION 1

1. (a) Find all the roots of the polynomial

$$x^3 - 6x^2 + 11x - 6 = 0$$

[8 marks]

- (b) Determine the centre and radius of the circle

$$x^2 + y^2 - 6x + 8y - 11 = 0$$

[8 marks]

- (c) Find an equation of the line perpendicular to the line $y + 2x = 3$ and passing through the point (1,3)

[4 marks]

QUESTION 2

2. (a) Solve each of the following equations for x

i. $3^{2x-3} = 243$

[6 marks]

ii. $\log_3(x+6) - \log_3(x+2) = \log_3 x$

[6 marks]

- (b) Expand and simplify $(2x + 3y)^5$

[8 marks]

QUESTION 3

3. (a) Find the first four terms in the expansion of $(1 - x)^{-1}$.

[6 marks]

- (b) Prove the following identities

i. $\tan A + \cot A = \sec A \csc A$

[4 marks]

ii. $(1 - \cos A)(1 + \sec A) = \sin A \tan A$

[4 marks]

- (c) The population of Mbabane varies according to the equation

$$P = 100000e^{0.15t}$$

where t is time in years. Find the time it will take for the population to double. [6 marks]

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QUESTION 4

4. (a) Use Cramer's rule to solve the following linear system of equations

$$x + 3y + 3z = 16$$

$$x + 4y + 3z = 18$$

$$2x + 7y + 7z = 37$$

[12 marks]

- (b) Solve the following trigonometric equations giving all solutions between 0° and 360°

$$2\cos^2 x - \sin x - 1 = 0$$

QUESTION 5

5. (a) Use the limit definition of the derivative to find $f'(x)$, given that

$$f(x) = x^2 + x$$

[10 marks]

- (b) Find the derivative of $y = (x^2 + x + 1)^3$

4 marks

- (c) Find the second derivative, y'' , of the function $y = (2x + 1)^8$

[6 marks]

QUESTION 6

6. (a) Evaluate the following integrals

i. $\int (1 + 2x^2 + \sqrt{x}) dx$

[5 marks]

ii. $\int \frac{9x^2}{(x^3 + 1)^4} dx$

[5 marks]

- (b) Find the area enclosed by the curve $y = 9 - x^2$ and the x -axis.

[10 marks]

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QUESTION 7

7. (a) For the function, $y = x^3 - 6x^2 + 9x + 1$ find and classify all stationary points, determine intervals of increase and decrease and sketch its graph. [10 marks]
- (b) A cattle owner has 800 metres of fence which he wishes to use to make a rectangular holding region in which his cattle will graze. IF the region will border an existing fence, find the dimensions of the region and calculate the largest area that can be enclosed. [10 marks]

QUESTION 8

8. (a) Differentiate the following functions
- i. $y = (x + 1)\sqrt{x^2 + 1}$ [5 marks]
- ii. $y = \frac{x^2 - 1}{x^2 + 1}$ [5 marks]
- (b) Integrate the following functions
- i. $\int 15x^2(x^3 - 1)^4 dx$ [5 marks]
- ii. $\int (1 + x + x^2 + x^3) dx$ [5 marks]