University of Swaziland

Final Examination April/May 2008

Dip. Env. Health I, Dip. Env. Health IV

Title of Paper : Calculus for Health Sciences

Course Number : HSM115

Time Allowed : Two (2) hours

Instructions :

1. This paper consists of SIX questions.

2. Each question is worth 25%.

3. Answer ANY FOUR questions.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find y' and simplify, given

(i) $y = 2x^2 - 4 - \dfrac{1}{x} + 2\sqrt{x}$ [5 marks]

(ii) $y = \ln 2 + \ln x - e^{-2x} - \cos 4x$ [5 marks]

(iii) $F(x) = \sqrt{1 + 2x^2}$ [5 marks]

(b) Integrate

(i) $\displaystyle\int_{-2}^{1} \left(2 - 2x + 3x^2\right) dx$ [5 marks]

(ii) $\displaystyle\int_{1}^{9} \left(3\sqrt{x} - \dfrac{1}{2\sqrt{x}}\right) dx$ [5 marks]

Question 2

(a) Find the indicated derivative

(i) $y = \left(8 - 4x + x^2\right)e^{\frac{1}{2}x}$, y' [6 marks]

(ii) $P(x) = 2x^3 \ln x$, $P'''(x)$ [6 marks]

(b) Use the method of partial fractions to evaluate

$$\int \frac{x + 4}{x^2 - x - 6} \, dx.$$ [13 marks]

Question 3

(a) Define an inflexion point of a graph, and describe/illustrate the two types of inflexion points. [5 marks]

(b) The number of cockroaches (in thousands) in the kitchen of a commercial food outlet is given by

$$C(t) = 40 - 6t^2 + t^3,$$

where t is the number of days after implementing a pest control treatment.

(i) When is the cockroach population increasing? Decreasing? [4marks]

(ii) When is the cockroach population stationary? Classify this stationary and find the number of cockroaches at this point. [6marks]

(iii) When does the population have an inflexion point? [4 marks]

(iv) Sketch the graph of $C(t)$. [6 marks]

Question 4

(a) A college of 6,000 students initially has no HIV positive students. After one student contracts the virus from outside, it is determined that the rate of infection among the students is given by

$$N'(t) = 600e^{-0.1t},$$

where $N(t)$ is the number of students infected t months later.

(i) Find the number of students infected during the *first* month. [5 marks]

(ii) What is the number of students infected during the *tenth* month. [4 marks]

(iii) How long does it take for 50% of the college population to be infected? [6 marks]

(b) Evaluate

$$\int \frac{6x}{\sqrt{4 + x^2}}\, dx.$$ [10 marks]

Question 5

(a) Suppose that, in evaluating $\lim_{x \to 2} f(x)$ where $f(x)$ is some given function, you find that direct substitution gives $\frac{16}{0}$. You then ask your friend who says "If direct substitution gives $\frac{16}{0}$, then it means that $\lim_{x \to 2} f(x) = \infty$."

Is your friend's statement right or wrong? Explain. [5 marks]

(b) Evaluate

(i) $\lim_{x \to \infty} \left(\frac{1 - 2x^2}{2x^2 - 101x + 50000} \right)$ [5 marks]

(ii) $\int_1^e \frac{x + 2}{x}\, dx$ [5 marks]

(b) Differentiate and simplify

$$y = \ln\left(\frac{1 + x}{1 - x} \right)$$ [10 marks]

Question 6

(a) Differentiate
$$G(\lambda) = \frac{\lambda^2}{2 - \lambda^2}$$
[7 marks]

(b) Integrate
$$\int x^2 e^{2x} \, dx$$
[8 marks]

(c) Find the area of the region bounded by the curves $y = x^2$ and $y = x + 12$.
[10 marks]

336