University of Swaziland

Final Examination, December 2008

Dip. Env. Health I, Dip. Env. Health IV

Title of Paper : Algebra for Health Sciences

Course Number : HSM111

Time Allowed : Two (2) hours

Instructions :

1. This paper consists of SIX questions.

2. Each question is worth 25%.

3. Answer ANY FOUR questions.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Express

$$\log \left(\frac{a^4 b^2}{c^6} \right)^{-\frac{1}{2}}$$

in terms of $\log a$, $\log b$ and $\log c$.   [4 marks]

(b) Find the term involving x in the expansion of $\left(2x - \frac{1}{\sqrt{x}} \right)^{16}$.

[5 marks]

(c) Find all real roots of $x^3 + x^2 - 4x + 4 = 0$. [10 marks]

(d) Find the centre and radius of the circle

$$x^2 + y^2 - 10x + 8y + 5 = 0.$$ [6 marks]

Question 2

(a) Solve for x

   i. $4^x = 3^{1-2x}$ [4 marks]

   ii. $\log_2 (x + 2) = 3 - \log_2 x$ [4 marks]

(b) Find the first five terms of the expansion of $\left(1 - 2x^2 \right)^{\frac{1}{2}}$.

[8 marks]

(c) Find 3 numbers in arithmetic progression such that their sum is 15 and their product is 80.   [9 marks]

Question 3

(a) The population of a city is given by

$$P(t) = 45,000e^{0.03t}$$

where t is the number of years from the year 2000.

i. Find the population in the year 2010.   [3 marks]

ii. When will the population be double that in 2000?
       [7 marks]

(b) Use Cramer's rule to solve for x, y and z, given

$$\begin{aligned}
x + y - z &= 7 \\
4x - y + 5z &= -4 \\
2x + 2y - 3z &= 0.
\end{aligned}$$

[15 marks]

Question 4

(a) Given that $\sin\theta = \frac{3}{5}$ and that θ is in QII, find

   i. $\sin 2\theta$ [5 marks]

   ii. $\cos 2\theta$ [5 marks]

   Hence state the quadrant in which 2θ lies. [2 marks]

(b) Evaluate

$$\frac{x^4 - x^3 + x + 2}{x^2 - 2}.$$
       [8 marks]

(c) Find the value of the infinite sum

$$18 - 12 + 8 - \cdots .$$ [5 marks]

Question 5

(a) Use the quadratic formula to solve

$$x^2 - 2x + 10 = 0.$$ [6 marks]

(b) Evaluate

$$\frac{2 + 2i}{(1 - i)^2} + \frac{2i}{i - 1}$$

and express your answer in the form $a + ib$. [7 marks]

(c) Find the value of the finite sum

$$1 - 2 + 2^2 - 2^3 + \cdots + 2^{10}.$$ [6 marks]

(d) Prove

$$\frac{\sec A + \csc A}{\tan A + \cot A} = \sin A + \cos A.$$ [6 marks]

Question 6

(a) Evaluate

$$\begin{vmatrix} 4 & 5 & 0 & -8 \\ 2 & -3 & 4 & 0 \\ 3 & 0 & -2 & 0 \\ 0 & 2 & 0 & -6 \end{vmatrix}.$$

[12 marks]

(b) Use de Moivre's theorem to evaluate

$$\left(\sqrt{3} - i\right)^6$$

and express your answer in the form $a + ib$.   [8 marks]

(c) Use synthetic division to evaluate

$$(x^4 - 3x^2 + 2x + 10) \div (x + 4).$$ [5 marks]