University of Swaziland

Final Examination, December 2011

Dip. Env. Health I, Dip. Env. Health IV

Title of Paper : Algebra for Health Sciences
Course Number : HSM111/EHM106
Time Allowed : Two (2) hours
Instructions :

1. This paper consists of SIX questions.
2. Each question is worth 25%.
3. Answer ANY FOUR questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Express

$$\ln\left(abc^2\right) - 4\ln\left(a\sqrt{b}\right) + 2\ln\left(\frac{a^2b}{c}\right)$$

as a single logarithm with coefficient 1. [4 marks]

(b) Find the middle term in the binomial expansion of

$$\left(x - \frac{2}{\sqrt{x}}\right)^{18}. \qquad [5 \text{ marks}]$$

(c) Find all real roots of $x^3 + x^2 - 4x - 4 = 0$. [10 marks]

(d) Find the centre and radius of the circle

$$x^2 + y^2 + 4x - 6y - 3 = 0. \qquad [6 \text{ marks}]$$

Question 2

(a) Solve for x

i. $3^x = 4^{3-2x}$ [4 marks]

ii. $\log_7(x+1) = 1 - \log_7(x-5)$ [8 marks]

(b) Find the first four terms of the binomial expansion of

$$(1-x)^{-2}. \qquad [8 \text{ marks}]$$

(c) Find the sum of the first 21 terms of the arithmetic progression

$$100,\ 92,\ 84,\ 76, \cdots. \qquad [5 \text{ marks}]$$

Question 3

(a) The population of a city is given by

$$P(t) = 86,000e^{0.026t}$$

where t is the number of years from the year 2000.

i. Find the population in the year 2011. [3 marks]

ii. When will the population be double that in 2000? [7 marks]

(b) Use Cramer's rule to solve for x, y and z, given

$$\begin{aligned} x + 2y - z &= -7 \\ 4x - y + 3z &= 13 \\ 2x + 2y - 5z &= -14. \end{aligned}$$

[15 marks]

Question 4

(a) Given that $\cos\theta = \frac{3}{5}$ and that θ is in QIV, find

i. $\sin\theta$ [3 marks]

ii. $\cos 2\theta$ [6 marks]

(b) Evaluate

$$\frac{x^5 - x^4 + 3x + 2}{x^2 + 2}.$$ [11 marks]

(c) Find the value of the infinite sum

$$125 - 100 + 80 - 64 + \cdots.$$ [5 marks]

Question 5

(a) Use the quadratic formula to solve

$$x^2 + 4x + 13 = 0. \qquad \text{[6 marks]}$$

(b) Find the value of

$$\frac{13i}{3-4i} + \frac{13i}{3+4i}$$

and express your answer in the form $a + ib$.

[7 marks]

(c) The second term of a GP is 12. If the fourth term is 192, find the first 3 terms. [6 marks]

(d) Prove

$$\sin A + \frac{\cos^2 A}{1+\sin A} = 1. \qquad \text{[6 marks]}$$

Question 6

(a) Consider the triangle ABC with vertices $A(-4,-3)$, $B(-2,1)$ and $C(0,-5)$.

i. Find the lenghts of the sides of the triangle.

[4 marks]

ii. Hence determine what type of triangle ABC is.

[6 marks]

(b) Use synthetic division to evaluate

$$(x^4 - 4x^2 + 2x - 12) \div (x-3). \qquad \text{[8 marks]}$$

(c) Given

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 0 & -4 \end{pmatrix},$$

Work out AA^T. [7 marks]