
University of Swaziland



Final Examination, December 2012

BSc Env. Health Sc. I

Title of Paper : Algebra for Health Sciences

Course Number : EHM106

Time Allowed : Two (2) hours

Instructions :

1. This paper consists of SIX questions.
2. Each question is worth 25%.
3. Answer ANY FOUR questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

- (a) Consider the arithmetic progression

$$20, 26, 32, \dots$$

Find the

- i. formula for the n -th term [3 marks]
ii. sum of the first 20 terms [4 marks]
- (b) Work out and express your answer in the form $a + ib$
- i. $(1 + 2i)(1 + 3i)(1 + 4i)$ [6 marks]
ii. $\frac{10}{1 + 3i} + \frac{10}{1 - 2i}$ [6 marks]
- (c) Find the centre and radius of the circle described by the equation

$$x^2 + y^2 - 8x + 6y + 16 = 0. \quad [6 \text{ marks}]$$

Question 2

- (a) Expand and simplify term by term

$$\left(3x^3 + \frac{y}{x}\right)^5 \quad [9 \text{ marks}]$$

- (b) Use de Moivre's theorem to expand

$$\left(1 + i\sqrt{3}\right)^6,$$

expressing your answer in the form $a + ib$. [9 marks]

- (c) A ball fall from a height of 20 metres. If the maximum height attained decreases by 4% each time, fin the total distance travelled by the ball in coming to rest. [7 marks]
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Question 3

(a) Solve for x

i. $\left(\frac{4}{9}\right)^{2x-3} = \frac{81}{16}$ [4 marks]

ii. $\log_3(4x - 7) = 2$ [3 marks]

iii. $\log_6 x + \log_6(x - 1) = 1$ [6 marks]

(b) Given the matrices

$$A = \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -4 \\ 2 & 5 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & -1 & 3 \\ 1 & 0 & 2 \end{pmatrix}$$

work out (where possible)

i. $2A - 3B^T$ [3 marks]

ii. $A^T B$ [3 marks]

iii. CB [3 marks]

iv. $C^T B$ [3 marks]

Question 4

(a) Express as a single logarithm with coefficient 1, and simplify

$$\log_a a^2 - 4\log_5 a^2 + 3\log_5 a^4 - 6\log_5 \sqrt{a}. \quad [9 \text{ marks}]$$

(b) Solve the system

$$\begin{aligned} x + 2y + z &= 0 \\ 2x - y + 2z &= 4 \\ 2x + y &= 1 \end{aligned}$$

using Cramer's rule.

[16 marks]

Question 5

(a) Given that $\sin A = \frac{3}{5}$ and A lies in QII , find the exact values of

i. $\cos 2A$ [5 marks]

ii. $\tan 2A$ [5 marks]

Hence state the quadrant in which the angle $2A$ lies. [2 marks]

(b) Find the 15th term in the binomial expansion of

$$\left(2x^2 - \frac{y}{x}\right)^{16}. \quad [5 \text{ marks}]$$

(c) The population of a city grows according to

$$P(t) = 160,000e^{0.02t}$$

where t is the number of years from year 2000. Find

i. the population of the city in year 2015; [3 marks]

ii. the date when the population will be double that in 2000. [5 marks]

Question 6

(a) Find the value of

i. $\sum_{n=10}^{107} (7n - 3)$ [4 marks]

ii. $\sum_{n=0}^{\infty} 60\left(\frac{2}{5}\right)^n$ [3 marks]

(b) Prove

$$1 - \frac{\cos^2 A}{1 + \sin A} = \sin A. \quad [9 \text{ marks}]$$

(c) Find a solution set of

$$2 \cos^2 \theta + \sin \theta = 2$$

in the interval $-180^\circ \leq \theta \leq 180^\circ$. [9 marks]
