University of Swaziland

Final Examination – November 2013

BSc in Environmental Sciences I

Title of Paper : Algebra for Health Sciences

Course Number : EHM106

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Give a concise definition of each of the following terms.

    i. A *polynmial* [2 marks]

    ii. The *transpose* of a matrix [2 marks]

    iii. The *common logarithm* of a number [2 marks]

  b. State *de Moivre's Theorem* [2 marks]

  c. Sketch the graph of $y = \ln x$. [2 marks]

A.2 a. Find the sum of the first 50 terms of the AP

$$100, 92, 84, \cdots .$$ [4 marks]

  b. Given the vectors $\underline{A} = 4\hat{i} - 4\hat{j} + 8\hat{k}$ and $\underline{B} = i + 2\hat{j} + \frac{1}{2}\hat{k}$, find

    i. $|\underline{A}|$ [4 marks]

    ii. $\underline{A} \cdot \underline{B}$ [4 marks]

  c. Given the matrices $A = \begin{pmatrix} 1 & -3 & 2 \\ -2 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & -1 \\ 3 & -2 \end{pmatrix}$, find the value

    of

$$B^T A^T.$$ [5 marks]

  d. Find the value of

    i. $\log_b b^{-5}$ [2 marks]

    ii. $\ln e^m$ [2 marks]

    iii. $(4 - 3i)(4i - 3)$ and express your answer in the form $x + iy$ [5 marks]

  e. Use synthetic division to find the quotient and remainder of

$$\frac{x^4 + 2x^3 - 13x + 21}{x + 3}.$$ [5 marks]

  f. Use the binomial theorem to expand the following expression and simplify term by term

$$\left(x + \frac{2}{x}\right)^5$$ [9 marks].

Section B
Answer ANY 2 Questions in this section

B.1 a. Use Cramer's rule to solve

$$\begin{aligned}
2x &- y &+ 3z &= 0 \\
x &+ 4y &- z &= 1 \\
5x & &+ 2z &= 17.
\end{aligned}$$

[18 marks]

b. Find the *exact* area of the triangle with vertices $A(1,1,1)$, $B(3,-3,3)$ and $C(-5,5,-5)$.

[7 marks]

B.2 a. Find the value of

i. $\displaystyle\sum_{n=-5}^{50}(22-8n)$

[6 marks]

ii. $\displaystyle\sum_{n=0}^{\infty}80\left(-\frac{3}{4}\right)^n$

[4 marks]

b. A seating section of a stadium has 45, 52 and 59 seats in the first, second and third rows, respectively. If the number of seats increase by 7 between successive rows, and the total number of rows is 19, find the total number of seats in the section.

[3 marks]

c. Given that $\sin\theta = \frac{\sqrt{3}}{2}$ and θ in in QII, find the *exact* values of

i. $\cos\theta$

[2 marks]

ii. $\cos 2\theta$

[4 marks]

d. Find the general solution of

$$3 - 4\cos^2\theta = 0.$$

[6 marks]

B.3

a. Find the quotient and remainder of the division

$$\frac{x^4 + 3x^3 - 2x + 3}{x^2 - 2}.$$

[9 marks]

b. Given that $x - 5$ is a factor of the polynomial $P(x) = x^3 + Ax^2 + Bx - 10$, while dividing $P(x)$ by $x + 3$ leaves a remainder of -16, find the values of A and B.

[5 marks]

c. Find the equation of a circle centred at $(-7, 9)$ tangent to the x-axis, and express it in general form.

[7 marks]

d. Solve

$$9x^2 - 12x + 20 = 0.$$

[4 marks]

B.4

a. Express as a single logarithm with coefficient of 1.

$$\ln\left(\frac{a^2}{bc}\right) + \ln\left(\frac{b^2}{ac}\right) + \ln\left(\frac{c^2}{ab}\right).$$

[4 marks]

b. Solve

$$\log_3 2x - \log_3(x - 7) = 2.$$

[4 marks]

c. In a college campus with a population of 2,000, the number of people infected by an air-borne infection is given by

$$P(t) = 2,000\left(1 - e^{-0.07t}\right),$$

where t is the number of days after the initial outbreak. Find the

i. number of people infected after 7 days [2 marks]

ii. number of days it takes for 75% of college to be infected. [4 marks]

d. For the binomial expansion of

$$\left(x^2 - \frac{y}{x^2}\right)^{20},$$

find the

i. first 4 terms [6 marks]

ii. the 18th term [3 marks]