University of Swaziland

Supplementary Examination – July 2014

BSc in Environmental Health Sciences I

Title of Paper   :   Algebra for Health Sciences

Course Number   :   EHM106

Time Allowed   :   Two (2) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY 2 questions in Section B.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Give a concise definition of each of the following terms.

     i.   A *polynomial*   [2 marks]

     ii. The *transpose* of a matrix   [2 marks]

     iii. The *modulus* of a complex number   [2 marks]

     iv. The *common logarithm* of a number   [2 marks]

  b. Sketch the graph of $y = e^x$.   [2 marks]

A.2 a. Find the formula of the n-th term for the AP

$$10, 22, 34, \cdots .$$ [4 marks]

  b. Given the vectors $\underline{A} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{B} = \hat{i} + 2\hat{j} - \frac{1}{2}\hat{k}$, find

     i. $|\underline{A}|$ [4 marks]

     ii. $\underline{A} \cdot \underline{B}$ [4 marks]

  c. Given the matrices $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & -2 \end{pmatrix}$, find the value of

$$3A - 2B^T.$$ [5 marks]

  d. Find the value of

     i. $\log_3 45, 210$ (correct to 3 s.f.)   [2 marks]

     ii. $\log 10^{-2m}$ [2 marks]

     iii. $3i(3 + 4i) - 4i(4 - 3i)$ and express your answer in the form $x + iy$ [5 marks]

  e. Use synthetic division to find the quotient and remainder of

$$\frac{x^4 + 16}{x + 2}.$$ [5 marks]

  f. Use the binomial theorem to expand the following expression and simplify term by term

$$(2x + 3y)^5$$ [9 marks].

Section B

Answer ANY 2 Questions in this section

B.1 a. Find the value of

$$\begin{vmatrix} 2 & 0 & -1 & 0 \\ -1 & 3 & 0 & 1 \\ 0 & -2 & 0 & -5 \\ 0 & 1 & 2 & 0 \end{vmatrix}$$

[10 marks]

b. Given the vectors $\underline{A} = 2\hat{i} - 4\hat{j} + 10\hat{k}$ and $\underline{B} = 4\hat{i} + 2\hat{j} - 2\hat{k}$, find

   i. the angle between the vectors $\underline{A}$ and $\underline{B}$ (correct to 1 d.p.)   [7 marks]

  ii. $\underline{A} \times \underline{B}$ [8 marks]

B.2 a. Find the value of

   i. $\displaystyle\sum_{n=5}^{50}(5n + 10)$ [6 marks]

  ii. $\displaystyle\sum_{n=0}^{\infty} 70\left(\frac{1}{2}\right)^n$ [4 marks]

b. A seating section of a stadium has 45, 52 and 59 seats in the first, second and third rows, respectively. If the number of seats increase by 7 between successive rows, and the total number of rows is 19, find the total number of seats in the section.   [3 marks]

c. Given that $P(5, -12)$ lies on the terminal side of the angle θ, find the *exact* values of

   i. $\cos\theta$ [3 marks]

  ii. $\sin\theta$ [3 marks]

d. Prove

$$\sec A - \cos A = \sin A \tan A.$$

[6 marks]

B.3

a. Find the quotient and remainder of the division

$$\frac{x^4 + x^3 - x + 3}{x^2 + 1}.$$

[9 marks]

b. Find the values of A and B such that both $x - 4$ and $x + 3$ are factors of $P(x) = x^3 + Ax^2 + Bx + 4$. [5 marks]

c. Expand

$$\left(1 - i\sqrt{3}\right)^3$$

and express your answer in the form $x + iy$, using

   i.   the binomial theorem [7 marks]

   ii. de Moivre's theorem [4 marks]

B.4

a. Express as a single logarithm with coefficient of 1.

$$4\ln(2\sqrt{x}) - \ln\left(4x^2\right).$$

[5 marks]

b. Solve

$$\log_8 2 + \log_8(x - 5) = 2.$$

[4 marks]

c. The book value of a machine is given by

$$B(t) = 25,000e^{-0.04t,}$$

where t is its age in years. Find its

   i.   value after years [2 marks]

   ii. half-life [5 marks]

d. Find the centre and radius of the circle defined by

$$x^2 + y^2 - 8x + 14y - 35 = 0.$$

Hence, make a sketch of the circle. [9 marks]