University of Swaziland

Final Examination – May 2014

BSc in Environmental Sciences I

Title of Paper : Calculus for Health Sciences

Course Number : EHM107

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. With the aid of graphical sketches and formulae, give a concise explanation of the term *relative maximum point* of a graph. [3 marks]

b. Explain the meaning of the term *indeterminate form* in the context of finding the limi

$$\lim_{x \to a} \frac{p(x)}{q(x)}.$$ [2 marks]

c. Evaluate

i. $\lim_{x \to -2} \left(\frac{3 - 2x}{7 - 4x - x^2} \right)$ [2 marks]

ii. $\lim_{x \to 2} \left(\frac{x - 2}{2x - x^2} \right)$ [4 marks]

iii. $\lim_{x \to \infty} \left(\frac{2x - 1}{x^2 + 2x - 1} \right)$ [3 marks]

A.2 a. State *limit definition* of the derivative of the function $f(x)$. [2 marks]

b. Use the limit definition to find $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ given

$$f(x) = 7x^2 - 3.$$ [7 marks]

c. Find y' if

i. $y = 1 - 2x^3 + x^6$ [2 marks]

ii. $y = \dfrac{x^3 - 2}{x^2}$ [3 marks]

iii. $y = 5X^{\frac{2}{5}} - 14X^{-\frac{3}{7}}$ [3 marks]

iv. $y = e^{2 - 3x^2}$ [2 marks]

v. $y = \ln(2 - 5x)$ [3 marks]

A.3 a. State the *Fundamental Theorem of Calculus*. [3 marks]

b. Integrate

i. $\displaystyle\int_{-2}^{4} \left(1 - 4x + 3x^5 \right) \mathrm{d}x$ [5 marks]

ii. $\displaystyle\int \left(\frac{7}{x} - \frac{8}{x^5} \right) \mathrm{d}x$ [3 marks]

iii. $\displaystyle\int \sin 2x \, \mathrm{d}x$ [3 marks]

Section B

Answer ANY 2 Questions in this section

B.4 a. Find the value of the limit

$$\lim_{x \to 0} \frac{2 - \sqrt{3x + 4}}{x}$$ [7 marks]

b. Find the indicated derivative

    i. $y = x\sqrt{2x - 1}$ y' [4 marks]

    ii. $y = \dfrac{e^x}{1 - e^x},$ y' [5 marks]

    iii. $y = x^5 - \dfrac{2}{x},$ y^{iv} [4 marks]

    iv. $y = x \cos 2x,$ y''' [5 marks]

B.5 a. Consider the function

$$y = 4x^2 + \frac{1}{x}.$$

    i. Find the equation of the *tangent* to the curve of y when $x = 1$. [7 marks]

    ii. Find the *stationary point* of the function and determine its nature.

                                             [6 marks]

b. A retail shop has determined that the price demand function for its product is

$$p = 18 - 0.0015x$$

where p is the unit price and x is the number of units sold per week. If the cost of each units is E3 which the weekly fixed costs stand at E7,500, find

    i. the number of units to sell to obtain the maximum profit per week

                                             [9 marks]

    ii. the maximum weekly profit.   [3 marks]

B.6 a. Evaluate

     i. $\displaystyle\int \frac{24x}{\left(x^2+9\right)^5}\,\mathrm{d}x$ [7 marks]

     ii. $\displaystyle\int 32x^2 \sin 2x\,\mathrm{d}x$ [8 marks]

b. Find the area of the region enclosed by the parabola $y = x^2$ and the straight line $y = x + 20$.   [10 marks]

B.7 a. Use the method of partial fractions to evaluate

$$\int \frac{x+7}{x^2 - x - 6}\,\mathrm{d}x.$$ [10 marks]

b. A company needs a *closed* rectangular box with a capacity of 1,000 cm^3 such that the length of its base is twice the width. Find the dimensions of such a box that will require the least amount of material to construct.   [15 marks]