University of Swaziland

Supplementary Examination – July 2015

BSc in Environmental Sciences I

Title of Paper : Algebra for Health Sciences
Course Number : EHM106
Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. If the seventh and seventeenth terms of an AP are -21 and 43, respectively find

i. the first term and the common difference [5 marks]

ii. the sum of the first 20 terms

b. Expand using the binomial theorem

$$\left(x + \frac{3}{x}\right)^4. \quad [10 \text{ marks}]$$

c. Use *long division* to evaluate

$$\frac{x^4 - 2x^3 + 2x - 7}{x + 3}. \quad [5 \text{ marks}]$$

d. Find the equation of the straight line AB between $A(-3, 2)$ and $B(1, -6)$. [5 marks]

e. Solve for x given

i. $\log_3(2x - 1) = 2$ [5 marks]

ii. $2^x = 70$ [5 marks]

f. Given the matrices $A = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$, find the value of

i. $3A^T - 4B$ [5 marks]

ii. $A^T B$ [5 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Use Cramer's rule to solve

$$\begin{aligned} 2x - y + z &= 3 \\ 2y - z &= 1 \\ -x + y &= 1. \end{aligned}$$ [15 marks]

b. Given the vectors $\underline{A} = 4\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\underline{B} = 3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$, find

i. $|\underline{A}|$ [5 marks]

ii. $\underline{A} \cdot \underline{B}$ [5 marks]

B.2 a. Find the value(s) of x such that the sequence

i. $8x + 4, 6x - 2, 2x - 7$ is an AP [3 marks]

ii. $x - 1, 2x - 2, 3x - 1$ [7 marks]

b. Insert 3 arithmetic means between -18 and 4. [3 marks]

c. Given that $\tan A = \sqrt{3}$ where A is in $QIII$, find the *exact* values of

i. $\sin A$ [3 marks]

ii. $\cos A$ [3 marks]

d. Prove that

$$\frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta} = 2\sec^2\theta.$$ [5 marks]

B.3

a. Work out and express in the form $a + ib$.

i. $(2+3i)(3-2i)$ [3 marks]

ii. $\dfrac{3-4i}{4+3i}$ [4 marks]

iii. $(1-i\sqrt{3})^6$ (using de Moivre's theorem) [6 marks]

b. Find all the roots of the equation

$$x^3 - 3x^2 + 4 = 0.$$ [6 marks]

c. Find the equation the circle centre $(-3, 2)$ passing through the point $(7, -5)$. [6 marks]

B.4

a. Solve

$$\ln(9-2x) - \ln(3x-1) = 0.$$ [8 marks]

b. The population of a city grows according to

$$P(t) = 250,000e^{0.09t},$$

where t is the number of years the year 2000. Find the

i. population of the city in 2015 [2 marks]

ii. date when the population will reach double that in year 2000. [5 marks]

c. For the binomial expansion of

$$\left(x + \frac{1}{x^2}\right)^{20},$$

find the

i. first 4 terms [6 marks]

ii. the 19th term [4 marks]