
University of Swaziland



Final Examination – May 2015

BSc in Environmental Sciences I

Title of Paper : Calculus for Health Sciences

Course Number : EHM107

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN
BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 Evaluate

i. $\lim_{x \rightarrow 3} \left(\frac{2x + 9}{x^2 - 3} \right)$ [2 marks]

ii. $\lim_{x \rightarrow -1} \left(\frac{2x^2 + x - 1}{4x^2 + 5x + 1} \right)$ [4 marks]

iii. $\lim_{x \rightarrow \infty} \left(\frac{7 + 2x - 12x^2}{3x^2 + 2x - 13} \right)$ [3 marks]

A.2 a. State *limit definition* of the derivative of the function $f(x)$. [2 marks]

b. State three (3) properties of the derivative. [3 marks]

c. Use the limit definition to find $\frac{df}{dx}$ given

$$f(x) = 7 - 2x^2. \quad [7 \text{ marks}]$$

d. Find y' if

i. $y = x - 4x^2 + 5x^4$ [2 marks]

ii. $y = \frac{2x^3}{3} - \frac{3}{2x^2}$ [3 marks]

iii. $y = 6X^{\frac{2}{3}} - 10X^{-\frac{7}{5}}$ [3 marks]

iv. $y = e^{2x} - e^{-3x}$ [2 marks]

v. $y = \ln x^{-\frac{2}{7}}$ [3 marks]

A.3 a. State the *Fundamental Theorem of Calculus*. [3 marks]

b. Integrate

i. $\int_{-1}^3 (3 - 2x + 4x^3) dx$ [5 marks]

ii. $\int \left(5X^{\frac{2}{3}} - \frac{4}{X} \right) dX$ [3 marks]

iii. $\int (e^{2x} - \sin 2x) dx$ [3 marks]

Section B

Answer ANY 2 Questions in this section

B.4 a. Find $\frac{df}{dx}$ using the *limit definition*, for

$$f(x) = \frac{1}{3 - 2x} \quad [7 \text{ marks}]$$

b. Find the indicated derivative

i. $y = (1 - 2x)e^{-2x}$ y' [4 marks]

ii. $y = \frac{x^2 + 4}{x^2 - 4}$, y' [5 marks]

iii. $y = x^4 - \frac{32}{\sqrt{x}}$, y^{iv} [4 marks]

iv. $y = x^3 \ln x$, y''' [5 marks]

B.5 a. Consider the function

$$y = 2x^3 + 3x^2 - 36x.$$

- i. Find the stationary points of y and determine their nature. [10 marks]
 - ii. Find the intervals where y is increasing/decreasing. [3 marks]
 - iii. Find the intervals where y is concave up/down. [3 marks]
 - iv. Find the inflexion point(s) of y . [4 marks]
 - v. Make a sketch of the graph of y clearly showing the stationary point(s), inflexion point(s) and the y -intercept. [5 marks]
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B.6 a. Evaluate

i. $\int \frac{24x}{\sqrt{x^2 + 5}} dx$ [7 marks]

ii. $\int 32x^2 e^{-2x} dx$ [9 marks]

b. The concentration of alcohol (in percentage) in the bloodstream is given by

$$P(t) = 0.4te^{-t/3}$$

where t is the number of hours after consumption. Find the

i. the time at which the concentration reaches a maximum in the blood. [5 marks]

ii. the maximum concentration. [4 marks]

B.7 a. The rate of seepage of toxic chemicals (in thousands of litres per year) from a dumping site is given by

$$R'(t) = \frac{4}{(1+t)^3}$$

where t is the number of years after discovering the seepage. Find the total volume of toxic chemicals that seep during the first 10 years. [6 marks]

b. Use partial fractions to integrate

$$\int \frac{dx}{2x^2 - x} \quad [12 \text{ marks}]$$

c. Find the area of the region enclosed by the curves

$$xy = 12, y = 0, x = 1, x = e^6.$$

[7 marks]
