University of Swaziland

Supplementary Examination – July 2015

BSc in Environmental Sciences I

Title of Paper : Calculus for Health Sciences

Course Number : EHM107

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY 2 questions in Section B.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 Evaluate

    i. $\lim\limits_{x \to 4} \left(\dfrac{3x + 4}{x^2 - 4} \right)$ [2 marks]

    ii. $\lim\limits_{x \to 2} \left(\dfrac{2 - x}{x^2 - 4} \right)$ [4 marks]

    iii. $\lim\limits_{x \to \infty} \left(\dfrac{2x - 7}{x^2 + 23} \right)$ [3 marks]

A.2 a. State *limit definition* of the derivative of the function $f(x)$.   [2 marks]

    b. State three (3) properties of the derivative.   [3 marks]

    c. Use the limit definition to find $\dfrac{df}{dx}$ given

$$f(x) = 9 - 4x.$$ [7 marks]

    d. Find y' if

        i. $y = 3 - 4x + 3x^2$ [2 marks]

        ii. $y = 8\sqrt{x} - \dfrac{7}{x}$ [3 marks]

        iii. $y = 10X^{\frac{2}{5}} - 6X^{-\frac{1}{3}}$ [3 marks]

        iv. $y = \sin 2x - e^{-4x}$ [2 marks]

        v. $y = \ln(2x - 1)$ [3 marks]

A.3 a. State the *Fundamental Theorem of Calculus*.   [3 marks]

    b. Integrate

        i. $\displaystyle\int_{-3}^{4} \left(5 + 4x^3 \right) dx$ [5 marks]

        ii. $\displaystyle\int \left(5\sqrt{X} + \dfrac{7}{X} \right) dX$ [3 marks]

        iii. $\displaystyle\int \dfrac{dx}{2x - 3}$ [3 marks]

Section B
Answer ANY 2 Questions in this section

B.4 a. Evaluate

$$\lim_{x \to 0} \frac{4 - \sqrt{x + 16}}{x}.$$ [7 marks]

   b. Find the indicated derivative

   i. $y = xe^{-2x}$ y' [4 marks]

   ii. $y = \dfrac{2x}{3 - 5x}$, y' [5 marks]

   iii. $y = x^3 - \dfrac{2}{x}$, y''' [4 marks]

   iv. $y = \cos 2x$, y^{iv} [5 marks]

B.5 a. Consider the function

$$y = 4 + 3x^2 - x^3.$$

   i. Find the stationary points of y and determine their nature.   [10 marks]

   ii. Find the intervals where y is increasing/decreasing.   [3 marks]

   iii. Find the intervals where y is concave up/down.   [3 marks]

   iv. Find the inflexion point(s) of y.   [4 marks]

   v. Make a sketch of the graph of y clearly showing the stationary point(s), inflexion point(s) and the y-intercept.   [5 marks]

B.6 a. Evaluate

   i. $\displaystyle\int \frac{dx}{\left(x^2 + 5\right)^3}$ [7 marks]

   ii. $\displaystyle\int 4xe^{-2x}\, dx$ [9 marks]

   b. The concentration of alcohol (in percentage) in the bloodstream is given by

$$P(t) = 0.4te^{-t/3}$$

   where t is the number of hours after consumption. Find the

   i. the time at which the concentration reaches a maximum in the blood. [5 marks]

   ii. the maximum concentration. [4 marks]

B.7 a. The rate of seepage of toxic chemicals (in thousands of litres per year) from a dumping site is given by

$$R'(t) = \frac{10}{(1 + t)}$$

   where t is the number of years after discovering the seepage. Find the total volume of toxic chemicals that seep during the first 10 years. [6 marks]

   b. Use partial fractions to integrate

$$\int \frac{dx}{x^2 - x - 6}.$$ [12 marks]

   c. Find the area of the region enclosed by the curves

$$y = x^2, \; y = x + 12.$$

[7 marks]