University of Swaziland

Final Examination – December 2015

BSc in Environmental Sciences I

Title of Paper	:	Algebra for Health Sciences
Course Number	:	EHS101
Time Allowed	:	Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. Find the *sum of the first 60 terms* of each number sequence

i. $50,\ 55,\ 60,\ 65,\ 70,\ \cdots$ [4 marks]

ii. $50,\ 55,\ 60.5,\ 66.55,\ 73.205,\ \cdots$ [4 marks]

b. Find the first 4 terms of the binomial expansion of

$$\left(x^2 + \frac{1}{x}\right)^{15}.$$ [6 marks]

c. Evaluate and leave your answer in the form $a + ib$.

i. $2i^7(3+5i) - 3i(2-3i)$ [4 marks]

ii. $\dfrac{2-3i}{3+2i}$ [4 marks]

d. Find the equation of a circle centres at $(4, -5)$ with radius $\sqrt{7}$, leaving your answer in *general form.* [5 marks]

e. Find the value of (express non-exact answers correct to s.f.)

i. $\log 4700$ [1 mark]

ii. $\ln 4700$ [1 mark]

iii. $\log_2 4700$ [1 mark]

v. $\log 1000^n$ [2 marks]

vi. $\ln e^{2n+1}$ [2 marks]

vii. $\log_b \left(\dfrac{1}{b^n}\right)$ [2 marks]

f. Given the matrices $A = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, find the value of

i. $A^T B$ [5 marks]

ii. $B^T B$ [5 marks]

g. Given that $\tan\theta = -\frac{5}{12}$ and $\cos\theta$ is negative, find the exact value of $\sin\theta$. [4 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Use Cramer's rule to solve

$$\begin{aligned} 3x - 2y + z &= 0 \\ 2x + y - z &= 13 \\ x - 4y &= -5. \end{aligned}$$

[16 marks]

b. Given the vectors $\underline{A} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\underline{B} = -3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$, find

i. $|2\underline{A} - 3\underline{B}|$ [6 marks]

ii. $\underline{A} \cdot \underline{B}$ [3 marks]

B.2 a. Find the value of

i. $\sum_{n=0}^{50}(7n - 2)$ [5 marks]

ii. $\sum_{n=0}^{19} 2^n$ [5 marks]

ii. $\sum_{n=0}^{\infty} 75\left(-\frac{2}{3}\right)^n$ [5 marks]

b. Find the value(s) of x such the the sequence

$$(2x - 5),\ (x - 4),\ (10 - 3x)$$

is a geometric progression. [7 marks]

c. After winning the lottery, a contestant is told that their payments will be as shown in the table below.

Beginning of month	1	2	3	4	···
Pay (in Emalangeni)	10,000	9,000	8,100	7,290	···

If the payments go on "forever", following the same trend,find the total amount that will be received by the contestant. [3 marks]

B.3

a. In the binomial expansion of

$$\left(x^2 - \frac{2y^2}{x^3}\right)^{20}$$

find

i. the 7th term [4 marks]

ii. the term involving x^{15} [6 marks]

b. Consider the polynomial

$$P(x) = 2x^3 - x^2 - 8x + 4.$$

i. Find the remainder when $P(x)$ is divided by

$x + 1$ [3 marks]

$x - 2$ [3 marks]

$x + 3$ [3 marks]

ii. Hence, or otherwise, factorise $P(x)$ and determine its roots. [6 marks]

B.4

a. Solve for x (expressing non-exact aswers correct to 3 s.f.)

i. $7^{2x-3} = 9000$ [6 marks]

ii. $\ln(3x - 2) = 0$ [6 marks]

iii. $\log(3x) - \log(9 - 0.7x) = 1$ [6 marks]

b. On 01 January 2015, a sum of E9,600 is invested in an account where it grows according to the formula

$$v(t) = 9,600e^{0.069t},$$

where t is the number of years after 01 January 2015. Find the

i. amount in the account on 31 December 2020 [2 marks]

ii. date at which the amount in the account will double. [5 marks]