University of Swaziland

Final Examination – May 2016

BSc in Environmental Sciences I

Title of Paper   :   Calculus for Health Sciences

Course Number :   EHS102

Time Allowed   :   Two (2) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY 2 questions in Section B.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 Evaluate

    i. $\displaystyle\lim_{x\to0}\left(e^{2x}+\ln(1+2x)\right)$ [2 marks]

    ii. $\displaystyle\lim_{x\to2}\left(\frac{2x-x^2}{x^2-4}\right)$ [4 marks]

    iii. $\displaystyle\lim_{x\to\infty}\left(\frac{5+3x}{x^2+2x-3}\right)$ [3 marks]

A.2   a. Use the limit definition to find $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ given

$$f(x)=\frac{1}{x}.$$ [7 marks]

    b. Find y' if

      i. $y=5x^3-\dfrac{7}{x^4}+5e^{-4x}+\ln(4x)$ [4 marks]

      ii. $y=\dfrac{2x}{3-4x}.$ [3 marks]

      iii. $y=\left(4x^3-1\right)^{20}$ [3 marks]

    c. Find y''' if

$$y=5x^4+64\sqrt{x}.$$ [3 marks]

A.3 Integrate

    i. $\displaystyle\int_1^9\left(3x^2-\frac{4}{\sqrt{x}}\right)\mathrm{d}x$ [5 marks]

    ii. $\displaystyle\int\frac{\mathrm{d}x}{3x-2}$ [3 marks]

    iii. $\displaystyle\int\left(2-6e^{-2x}+9\sin 3x\right)\mathrm{d}x$ [3 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Find the indicated derivative

     i. $y = \left(e^{2x} - e^{-2x}\right)^2$ y' [4 marks]

     ii. $y = \cos 2x - \sin 3x,$ y^{iv} [4 marks]

     iii. $y = (3x - 1)e^{3x},$ y''' [5 marks]

  b. An open-top box is to be constructed by cutting out small equal squares from the corners of a 90cm × 90cm square cardboard, and folding up. Find the dimensions of the box with largest volume that can be constructed.

                                                    [12 marks]

B.2 a. Consider the function

$$y = 10 + 36x - 3x^2 - 2x^3.$$

     i. Find the stationary points of y and determine their nature.   [10 marks]

     ii. Find the intervals where y is increasing/decreasing.   [3 marks]

     iii. Find the intervals where y is concave up/down.   [3 marks]

     iv. Find the inflexion point(s) of y.   [4 marks]

     v. Make a sketch of the graph of y clearly showing the stationary point(s), inflexion point(s) and the y-intercept.   [5 marks]

B.3   a. Evaluate

i. $\displaystyle\int \frac{\mathrm{d}x}{2x^2 + x}$ [7 marks]

ii. $\displaystyle\int x \ln x \, \mathrm{d}x$ [9 marks]

b. After the launch of a new cellphone, the rate of change of sales is found to be

$$S'(t) = \frac{6}{(1 + 2t)^{\frac{2}{3}}}$$

where t is the number of months after the launch and $S(t)$ is the number of sales in thousands. Find the total number of sales made

i.   during the first 6 months   [5 marks]

ii.   during the next 6 months.   [4 marks]

B.4   a. Integrate

i. $\displaystyle\int_0^{\sqrt{3}} \frac{x \, \mathrm{d}x}{\sqrt{x^2 + 1}}$ [9 marks]

ii. $\displaystyle\int 24x \sin 2x \, \mathrm{d}x$ [6 marks]

b. Find the area of the region enclosed by the curves
$y = x^2 - 3x - 9$ and $y = 1$.   [10 marks]