
University of Swaziland



Final Examination – December 2016

BSc in Environmental Sciences I

Title of Paper : Algebra for Health Sciences

Course Number : EHS101

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN
BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Find the value of the sum

i. $\sum_{n=-5}^{75} (5 - 8n)$ [5 marks]

ii. $\sum_{n=0}^{\infty} 75\left(\frac{4}{9}\right)^n$ [4 marks]

b. Given that $\tan \theta = \frac{4}{3}$ while $\sin \theta < 0$, find the *exact* value of $\cos \theta$. [5 marks]

c. Prove that

$$\cos^2 \theta (\sin^2 \theta + \cos^2 \theta + \tan^2 \theta) = 1. \quad [5 \text{ marks}]$$

c. Evaluate the complex number

i. $(3 - 5i)^2 - (3i + 5)^2$ [4 marks]

ii. $\frac{4i + 3}{4 - 3i}$ [4 marks]

and leave your answer in the form $a + ib$.

d. Find the equation of the straight line from $(4, -5)$ to $(-3, 9)$. [6 marks]

e. Solve for x (express non-exact answers correct to 2 d.p.)

i. $3^{2x-1} = 7439$ [4 marks]

ii. $\ln\left(\frac{4x-7}{2x+15}\right) = 0$ [5 marks]

f. Given the vectors $\mathbf{A} = 12\hat{i} - 16\hat{k}$ and $\mathbf{B} = 8\hat{i} - 2\hat{j} + 3\hat{k}$, find

i. $\mathbf{A} \cdot \mathbf{B}$ [2 marks]

ii. $\mathbf{A} \times \mathbf{B}$ [6 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Evaluate

$$\begin{vmatrix} 4 & 0 & -2 & 3 \\ 1 & 0 & 0 & -4 \\ 0 & -3 & 5 & 2 \\ 7 & 0 & 0 & 3 \end{vmatrix} \quad [10 \text{ marks}]$$

b. Use Cramer's rule to solve

$$\begin{aligned} 2x - y + 3z &= 0 \\ x + 2y - 2z &= 6 \\ 5x + 2y &= 12 \end{aligned} \quad [15 \text{ marks}]$$

B.2 a. Consider the triangle with vertices $A(4, 7)$, $B(-5, 2)$ and $C(6, -9)$. Find

- i. the perimeter of the triangle [6 marks]
- ii. the interior angle \hat{A} [4 marks]
- iii. the area of the triangle [6 marks]

b. A circle is centred at $C(-4, 7)$ and passes through the point $(-5, 2)$. Find

- i. the equation of the circle in *general form* [5 marks]
 - ii. the perimeter and area of the circle [2,2 marks]
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B.3 a. In the binomial expansion of

$$\left(x^2 - \frac{1}{x^3}\right)^{18}$$

find

- i. the 17th term [3 marks]
- ii. the term involving x^{-4} [7 marks]

b. Consider the polynomial

$$P(x) = 12x^3 + Ax^2 - 17x - 10,$$

where A is a constant. It is given that $x + 1$ is a factor of $P(x)$.

- i. Find the value of A [3 marks]
- ii. Hence, or otherwise, factorise $P(x)$ and determine its roots. [7 marks]

c. Use *synthetic division* to find the quotient and remainder of

$$\frac{x^4 - 2x^3 + 2x - 7}{x + 2}. \quad [5 \text{ marks}]$$

B.4 a. Solve for x (expressing non-exact answers correct to 2 d.p.)

- i. $3 \cdot e^{x-2} = 7^x$ [6 marks]
- ii. $\log_4(5x + 1) - \log_4(x + 7) = 1$ [6 marks]

b. On 01 January 2016, a sum of E7,500 is invested in an account which pays 9% interest, compounded daily. The amount grows according to the formula

$$A(t) = 7,500 \left(1 + \frac{0.09}{365}\right)^{365t},$$

where t is the number of years after 01 January 2016. Find the

- i. amount in the account on 01 July 2020 [2 marks]
- ii. date on which the amount in the account will reach E14,000. [6 marks]

c. The pH of an aqueous solution is given by $\text{pH} = -\log [H^+]$ where $[H^+]$ is the concentration of hydronium ions in the solution.

- i. Find the pH correct to 2 decimal places for lemon juice with $[H^+] = 8.46 \times 10^{-4}\text{M}$ [2 marks]
- ii. Find the concentration of hydronium ions correct to 3 significant figures for egg white with $\text{pH} = 8.27$ [3 marks]