
University of Swaziland



Final Examination – May 2017

BSc in Environmental Sciences I

Title of Paper : Calculus for Health Sciences

Course Number : EHS102

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 Evaluate

i. $\lim_{x \rightarrow 0} \left(\frac{x^2 + 10x}{x^2 - 4x} \right)$ [4 marks]

ii. $\lim_{x \rightarrow \infty} \left(\frac{3x^4 - 2x^2 + 7}{2000 + 3x^2 - 6x^4} \right)$ [3 marks]

A.2 a. Use the limit definition to find $\frac{df}{dx}$ given

$$f(x) = 4x^2 - 1. \quad [7 \text{ marks}]$$

b. Find y' if

i. $y = 4x^3 - \frac{2}{x^2} + 12\sqrt{x}$ [4 marks]

ii. $y = 3\pi - e^{-3x} + \sin 2x + \ln(2x)$ [3 marks]

iii. $y = \sqrt{5x^2 + 9}$ [3 marks]

c. Find y''' if

$$y = 2x^3 - \frac{2}{x^3}. \quad [3 \text{ marks}]$$

A.3 Integrate

i. $\int_1^{16} (12\sqrt{x} - 4x^3 + 10) dx$ [5 marks]

ii. $\int_1^5 \left(\frac{7}{x} - 15e^{-3x} - \frac{8}{x^3} \right) dx$ (correct to 2 d.p.) [6 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Consider the graph of the function

$$y = \ln \left(\frac{x+9}{3x-1} \right).$$

- i. Find the value(s) of x when $y = 0$. [4 marks]
- ii. Hence, find the equation of the tangent(s) and normal(s) at $y = 0$. [9 marks]

b. A farmer has E24,000 with which to construct a rectangular holding for her livestock. One of the sides is to use heavy-duty fence costing E40/metre while the other 3 will use regular fence costing E30/metre. Find the dimensions of the region enclosing the largest area that can be constructed.

[12 marks]

B.2 a. Differentiate and simplify

i. $F(x) = \frac{e^{-x}}{e^x - e^{-x}}$ [7 marks]

ii. $G(x) = \frac{1}{1-2x} + \ln(1-2x)$ [6 marks]

b. Consider the function

$$f(x) = 7 + 18x^2 - x^4.$$

- i. Find the stationary points of $f(x)$ and their nature. [6 marks]
 - ii. Find the inflexion points of $f(x)$ [2 marks]
 - iii. Make a sketch of the graph of $y = f(x)$. [4 marks]
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B.3 a. Evaluate

i. $\int_4^7 \frac{5x \, dx}{(x+2)(x-3)}$ [12 marks]

ii. $\int 8x^2 e^{-2x} \, dx$ [5 marks]

b. The rate of seepage of a toxic chemical (in litres per year) is given by

$$R'(t) = \frac{1000}{1+2t},$$

where t is the number of years from 01 Jan 2016. Find the *total* volume (to the nearest litre) of chemical that will have seeped by 31 December 2020.

[8 marks]

B.4 a. A bullet is fired vertically upwards at 343 m/s from the top of a 200m tower. Taking gravitational acceleration to be 9.8 m/s², find

i. the formula for the velocity of the bullet [4 marks]

ii. the formula for the height of the bullet [4 marks]

iii. the maximum height reached by the bullet [4 marks]

iv. the time at which the bullet hits the ground (in seconds, correct to 1 d.p.) [3 marks]

b. Find the area of the region bounded by the curves $y = 6x^2$ and $y = x + 2$.

[10 marks]

END OF EXAMINATION
