



UNIVERSITY OF SWAZILAND
Faculty of Health Sciences
Department of Environmental Health Science

DEGREE IN ENVIRONMENTAL HEALTH SCIENCE
FINAL EXAMINATION PAPER 2017

TITLE OF PAPER : STATISTICS FOR HEALTH SCIENCES
COURSE CODE : EHM 300
DURATION : 2 HOURS
MARKS : 100

INSTRUCTIONS : READ THE QUESTIONS & INSTRUCTIONS CAREFULLY
: ANSWER **ANY FOUR** QUESTIONS.
: EACH QUESTION **CARRIES 25** MARKS.
: WRITE NEATLY & CLEARLY
:
: BEGIN EACH QUESTION ON A SEPARATE SHEET OF PAPER.

DO NOT OPEN THIS QUESTION PAPER UNTIL PERMISSION IS GRANTED BY THE INVIGILATOR.

QUESTION ONE

(a) Define the following terms as applied in statistics:

- (i) Parameter [1 Mark]
- (ii) Discrete random variable [1 Mark]
- (iii) Gaussian distribution [1 Mark]
- (iv) Random variable [1 Mark]
- (v) Pulled variance [1 Mark]
- (vi) Sample size [1 Mark]
- (vii) Correlation coefficient [1 Mark]

(b) Table 1 below gives the ages of patients seen in emergency room of Mbabane Governmental hospital on Friday night.

Table 1. The ages of patients seen in emergency room of Mbabane Governmental hospital on Friday night

| | | | | | |
|----|----|----|----|----|----|
| 35 | 32 | 21 | 43 | 39 | 60 |
| 36 | 12 | 54 | 45 | 37 | 53 |
| 45 | 23 | 64 | 10 | 34 | 22 |
| 36 | 45 | 55 | 44 | 55 | 46 |
| 22 | 38 | 35 | 56 | 45 | 57 |

- (i) Using class width of 10, construct a frequency distribution and a relative frequency distribution for the data in Table 1. [6 Marks]
- (ii) Calculate the grouped mode of data in Table 1. [6 Marks]
- (iii) What is the median of grouped data in Table 1? [6 Marks]

Total [25 Marks]

QUESTION TWO

- (a) The score obtained on a test for anxiety neurosis is known to be normally distributed with a mean of 47 and standard deviation of 10. Find the probability that a person randomly selected from the population will score on this test:
 - (i) Less than 60,
 - (ii) Less than 40,
 - (iii) More than 45,
 - (iv) Between 45 and 60
 - (v) More than 58. [10 Marks]

- (b) During last year May 2016 final examination in Statistics, out of 50 students in class 40 students passed the examination. If you select 10 students at random what is the probability that exactly one student failed the examination? [15 Marks]

Total [25 Marks]

QUESTION THREE

- (a) What are the main difference between mode and standard deviation? [5 Marks]

- (b) The following are the systolic blood pressures (mm Hg) of 12 patients undergoing drug therapy for hypertension:

180, 150, 175, 155, 195, 160, 142, 112, 178, 152, 115, and 154.

Can we conclude on the basis of these data that the population mean is less than 165?

[20 Marks]

Total [25 Marks]

QUESTIONS FOUR

- (a) Briefly outline steps you could take in computing the Coefficient of Determination. **[5 Mark]**
- (b) A research was carried to compare different rainfall received in three (3) towns in Swaziland over a specified period (Table 2).

Table 2. Summary of rainfall recorded for the town of Mbabane, Manzini and Piggs Peak.

| | Average monthly rainfall) (mm) | | | |
|------------|--------------------------------|---------|----------|----------|
| | September | October | November | December |
| Mbabane | 68 | 145 | 175 | 205 |
| Manzini | 35 | 70 | 112 | 102 |
| Piggs Peak | 12 | 85 | 110 | 119 |

Using appropriate statistical tool carry out an analysis to check if there's any significance difference amongst the rainfall of three (3) towns listed: Mbabane, Manzini and Peak Piggs. **[20 marks]**

Total [25 Marks]

QUESTIONS FIVE

- (a) Briefly outline the components of the straight line regression model. **[5 Marks]**
- (b) The experiment to compare three methods of relieving stress was conducted. Each subject was placed in a stressful situation on three different occasions. Each time a different method for reducing stress was used with the subject. The response variable is the amount of decrease in stress level as measured before and after treatment application. The results were as follows:

Table 3. The response variable expressed as amount of decrease in stress level as measured before and after treatment.

| Treatment | | |
|-----------|----|----|
| A | B | C |
| 16 | 26 | 22 |
| 16 | 20 | 23 |
| 17 | 21 | 22 |
| 28 | 29 | 36 |

Can we conclude from these data that the three methods differ in effectiveness?

[20 Marks]

Total [25 Marks]

Course EHM 300:**Health Statistics formulae**UNGROUPED DATA

$$\text{Mean} = \frac{\sum x_i}{n}$$

$$\text{Median position: (n odd)} = x_{(n+1)/2}$$

$$\text{Median position (n even)} = \frac{x_{n/2} + x_{n/2+1}}{2}$$

$$\text{Variance} = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

$$\text{Standard deviation} = \sqrt{\left(\frac{\sum (x_i - \bar{x})^2}{n - 1} \right)}$$

GROUPED DATA

$$w = \frac{(x_L - x_s)}{1 + 3.322 \log_{10} n}$$

$$\text{Mean} = \frac{\sum m_i f_i}{n}$$

$$\text{Median } M_e = O_{me} + \frac{c[n/2 - f(<)]}{f_{me}}$$

$$\text{Mode } M_o = O_{mo} + \frac{c(f_m - f_{m-1})}{2f_m - f_{m-1} - f_{m+1}}$$

$$\text{Variance} = \frac{\sum (m_i - \bar{x})^2 f_i}{n - 1}$$

$$\text{Standard deviation} = \sqrt{\left(\frac{\sum (m_i - \bar{x})^2 f_i}{n - 1} \right)}$$

$$k^{\text{th}} \text{ percentile} = L_i + \frac{k/100 (n) - cf_{p(k-1)}}{f_{pk}} [U_i - L_i]$$

$$\text{Lower quartile (} Q_1) = L_i + \frac{(n+1)/4 - cf_{Q_1-1}}{f_{Q_1}} [U_i - L_i]$$

$$\text{Upper quartile (} Q_3) = L_i + \frac{3[(n+1)/4] - cf_{Q_3-1}}{f_{Q_3}} [U_i - L_i]$$

Confidence intervals for a:

A. a single population mean

$$\text{Population variance known} = \bar{x} \pm z_{1-\alpha/2} \sigma/\sqrt{n}$$

$$\text{Population variance unknown, (n>30)} = \bar{x} \pm t_{1-\alpha/2} s/\sqrt{n}$$

(population variance unknown)

B. Difference between two population means:

$$\text{Population variances known (i.e. } \mu_1 - \mu_2) = (\bar{x}_1 - \bar{x}_2) \pm z_{1-\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\text{Population variances unknown (i.e. } \mu_1 - \mu_2) = (\bar{x}_1 - \bar{x}_2) \pm t_{1-\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\text{C. Single Population Proportion (p): } = \hat{p} \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\text{D. Difference between two population proportions} = (\hat{p}_1 - \hat{p}_2) \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Regression and Correlation

$$b = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\frac{\sum x_i^2 - (\sum x_i)^2}{n}}$$

$$r = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

$$t = \frac{\hat{\beta}_1}{s/\sqrt{SS_{xx}}} \quad t_{0.025, (n-2)}$$

$$Y = \beta_0 + \beta_1 X \quad SSE = \sum (y_i - \bar{y})^2$$

$$\hat{\beta}_1 = SS_{xy}/SS_{xx} \quad SS_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

$$SS_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} \quad SS_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$s^2 = SSE/n-2$$

$$r^2 = \frac{SS_{yy} - SSE}{SS_{yy}} \quad r = \frac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}}$$

$$SS_{yy}$$

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Skewness formula (s), note that sign "s" in this context represent skewness and not standard deviation.

$$S = \sqrt{n} \frac{\sum_{i=1}^n (X_i - X_{avg})^3}{(\sum_{i=1}^n (X_i - X_{avg})^2)^{3/2}}$$

where X_{avg} , denote sample mean (\bar{x})

or

$$\text{Skewness} = \frac{\sqrt{n} \sum (x_i - \bar{x})^3}{(n-1)\sqrt{n-1} s^3}$$

$$\text{Kurtosis} = \frac{n \sum x_i - \bar{x} - 3}{(n-1)^2 s^4}$$

Quartile location in ordered array: $Q_1 = 1/4(n+1)$, **Interquartile range (IQR)** = $Q_3 - Q_1$

$$SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = SST - SSR = \sum_{i=1}^n Y_i^2 - b_0 \sum_{i=1}^n Y_i - b_1 \sum_{i=1}^n X_i Y_i$$

$Q_2 = 1/2(n+1)$, **Coefficient of variation (C.V)** = $s/\bar{x}(100)\%$

$Q_3 = 3/4(n+1)$

Sturges formular

$$k = 1 + 3.322(\log_{10}n)$$

Width of class intervals (w) = R/k

Range (R) = $x_L - x_S$

Standard normal distribution

$$Z = (x - \mu)/\delta$$

$$\text{Chi-square } X^2_w = \frac{\sum (f_{wi} - f_{thi})^2}{f_{thi}}$$

$$\text{Test statistics, } t_w = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$t_w = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2/n_1 + s^2/n_2}}$$

$$t_w = \bar{d}/s_{di}/\sqrt{n}$$

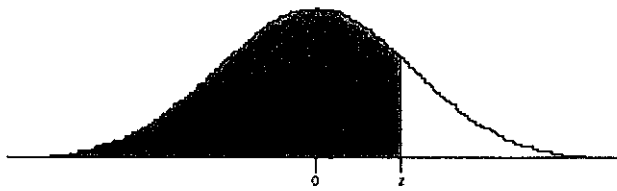
Pooled variance

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$b_1 = \frac{SSXY}{SSXX}$$

$$SSXY = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^n X_i Y_i - \frac{\left(\sum_{i=1}^n X_i\right)\left(\sum_{i=1}^n Y_i\right)}{n}$$

$$SSXX = \sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n}$$



| Normal Deviate z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -4.0 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.9 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.8 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.7 | .0001 | .0001 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.6 | .0002 | .0002 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 |
| -3.5 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 |
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| - .9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| - .8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| - .7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| - .6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| - .5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| - .4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| - .3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |

$$SSTotal = \sum \sum Y_{ij}^2 - T^2/n$$

$$SST = \sum T^2/r - T^2/n$$

$$SSE = SSTotal - SST$$



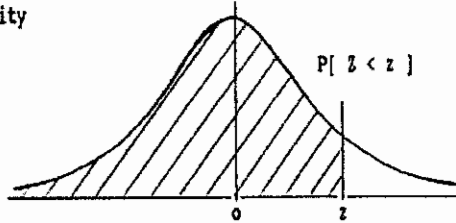
| Normal Deviate z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -4.0 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.9 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.8 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.7 | .0001 | .0001 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| -3.6 | .0002 | .0002 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 |
| -3.5 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 |
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[Z < z] = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}z^2) dz$$



| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

Upper-tail Critical values of chi-square distribution with ν degrees of freedom

| ν | Probability less than the critical value | | | | |
|-------|--|--------|--------|--------|--------|
| | 0.90 | 0.95 | 0.975 | 0.99 | 0.999 |
| 1 | 2.706 | 3.841 | 5.024 | 6.635 | 10.828 |
| 2 | 4.605 | 5.991 | 7.378 | 9.210 | 13.816 |
| 3 | 6.251 | 7.815 | 9.348 | 11.345 | 16.266 |
| 4 | 7.779 | 9.488 | 11.143 | 13.277 | 18.467 |
| 5 | 9.236 | 11.070 | 12.833 | 15.086 | 20.515 |
| 6 | 10.645 | 12.592 | 14.449 | 16.812 | 22.458 |
| 7 | 12.017 | 14.067 | 16.013 | 18.475 | 24.322 |
| 8 | 13.362 | 15.507 | 17.535 | 20.090 | 26.125 |
| 9 | 14.684 | 16.919 | 19.023 | 21.666 | 27.877 |
| 10 | 15.987 | 18.307 | 20.483 | 23.209 | 29.588 |
| 11 | 17.275 | 19.675 | 21.920 | 24.725 | 31.264 |
| 12 | 18.549 | 21.026 | 23.337 | 26.217 | 32.910 |
| 13 | 19.812 | 22.362 | 24.736 | 27.688 | 34.528 |
| 14 | 21.064 | 23.685 | 26.119 | 29.141 | 36.123 |
| 15 | 22.307 | 24.996 | 27.488 | 30.578 | 37.697 |
| 16 | 23.542 | 26.296 | 28.845 | 32.000 | 39.252 |
| 17 | 24.769 | 27.587 | 30.191 | 33.409 | 40.790 |
| 18 | 25.989 | 28.869 | 31.526 | 34.805 | 42.312 |
| 19 | 27.204 | 30.144 | 32.852 | 36.191 | 43.820 |
| 20 | 28.412 | 31.410 | 34.170 | 37.566 | 45.315 |
| 21 | 29.615 | 32.671 | 35.479 | 38.932 | 46.797 |
| 22 | 30.813 | 33.924 | 36.781 | 40.289 | 48.268 |
| 23 | 32.007 | 35.172 | 38.076 | 41.638 | 49.728 |
| 24 | 33.196 | 36.415 | 39.364 | 42.980 | 51.179 |
| 25 | 34.382 | 37.652 | 40.646 | 44.314 | 52.620 |
| 26 | 35.563 | 38.885 | 41.923 | 45.642 | 54.052 |
| 27 | 36.741 | 40.113 | 43.195 | 46.963 | 55.476 |
| 28 | 37.916 | 41.337 | 44.461 | 48.278 | 56.892 |
| 29 | 39.087 | 42.557 | 45.722 | 49.588 | 58.301 |
| 30 | 40.256 | 43.773 | 46.979 | 50.892 | 59.703 |
| 31 | 41.422 | 44.985 | 48.232 | 52.191 | 61.098 |
| 32 | 42.585 | 46.194 | 49.480 | 53.486 | 62.487 |
| 33 | 43.745 | 47.400 | 50.725 | 54.776 | 63.870 |
| 34 | 44.903 | 48.602 | 51.966 | 56.061 | 65.247 |
| 35 | 46.059 | 49.802 | 53.203 | 57.342 | 66.619 |
| 36 | 47.212 | 50.998 | 54.437 | 58.619 | 67.985 |
| 37 | 48.363 | 52.192 | 55.668 | 59.893 | 69.347 |
| 38 | 49.513 | 53.384 | 56.896 | 61.162 | 70.703 |
| 39 | 50.660 | 54.572 | 58.120 | 62.428 | 72.055 |
| 40 | 51.805 | 55.758 | 59.342 | 63.691 | 73.402 |

Upper-tail Critical values of chi-square distribution with ν degrees of freedom

| ν | Probability less than the critical value | | | | |
|-------|--|---------|---------|---------|---------|
| | 0.90 | 0.95 | 0.975 | 0.99 | 0.999 |
| 41 | 52.949 | 56.942 | 60.561 | 64.950 | 74.745 |
| 42 | 54.090 | 58.124 | 61.777 | 66.206 | 76.084 |
| 43 | 55.230 | 59.304 | 62.990 | 67.459 | 77.419 |
| 44 | 56.369 | 60.481 | 64.201 | 68.710 | 78.750 |
| 45 | 57.505 | 61.656 | 65.410 | 69.957 | 80.077 |
| 46 | 58.641 | 62.830 | 66.617 | 71.201 | 81.400 |
| 47 | 59.774 | 64.001 | 67.821 | 72.443 | 82.720 |
| 48 | 60.907 | 65.171 | 69.023 | 73.683 | 84.037 |
| 49 | 62.038 | 66.339 | 70.222 | 74.919 | 85.351 |
| 50 | 63.167 | 67.505 | 71.420 | 76.154 | 86.661 |
| 51 | 64.295 | 68.669 | 72.616 | 77.386 | 87.968 |
| 52 | 65.422 | 69.832 | 73.810 | 78.616 | 89.272 |
| 53 | 66.548 | 70.993 | 75.002 | 79.843 | 90.573 |
| 54 | 67.673 | 72.153 | 76.192 | 81.069 | 91.872 |
| 55 | 68.796 | 73.311 | 77.380 | 82.292 | 93.168 |
| 56 | 69.919 | 74.468 | 78.567 | 83.513 | 94.461 |
| 57 | 71.040 | 75.624 | 79.752 | 84.733 | 95.751 |
| 58 | 72.160 | 76.778 | 80.936 | 85.950 | 97.039 |
| 59 | 73.279 | 77.931 | 82.117 | 87.166 | 98.324 |
| 60 | 74.397 | 79.082 | 83.298 | 88.379 | 99.607 |
| 61 | 75.514 | 80.232 | 84.476 | 89.591 | 100.888 |
| 62 | 76.630 | 81.381 | 85.654 | 90.802 | 102.166 |
| 63 | 77.745 | 82.529 | 86.830 | 92.010 | 103.442 |
| 64 | 78.860 | 83.675 | 88.004 | 93.217 | 104.716 |
| 65 | 79.973 | 84.821 | 89.177 | 94.422 | 105.988 |
| 66 | 81.085 | 85.965 | 90.349 | 95.626 | 107.258 |
| 67 | 82.197 | 87.108 | 91.519 | 96.828 | 108.526 |
| 68 | 83.308 | 88.250 | 92.689 | 98.028 | 109.791 |
| 69 | 84.418 | 89.391 | 93.856 | 99.228 | 111.055 |
| 70 | 85.527 | 90.531 | 95.023 | 100.425 | 112.317 |
| 71 | 86.635 | 91.670 | 96.189 | 101.621 | 113.577 |
| 72 | 87.743 | 92.808 | 97.353 | 102.816 | 114.835 |
| 73 | 88.850 | 93.945 | 98.516 | 104.010 | 116.092 |
| 74 | 89.956 | 95.081 | 99.678 | 105.202 | 117.346 |
| 75 | 91.061 | 96.217 | 100.839 | 106.393 | 118.599 |
| 76 | 92.166 | 97.351 | 101.999 | 107.583 | 119.850 |
| 77 | 93.270 | 98.484 | 103.158 | 108.771 | 121.100 |
| 78 | 94.374 | 99.617 | 104.316 | 109.958 | 122.348 |
| 79 | 95.476 | 100.749 | 105.473 | 111.144 | 123.594 |
| 80 | 96.578 | 101.879 | 106.629 | 112.329 | 124.839 |
| 81 | 97.680 | 103.010 | 107.783 | 113.512 | 126.083 |

Upper-tail Critical values of chi-square distribution with ν degrees of freedom

| ν | Probability less than the critical value | | | | |
|-------|--|---------|---------|---------|---------|
| | 0.90 | 0.95 | 0.975 | 0.99 | 0.999 |
| 82 | 98.780 | 104.139 | 108.937 | 114.695 | 127.324 |
| 83 | 99.880 | 105.267 | 110.090 | 115.876 | 128.565 |
| 84 | 100.980 | 106.395 | 111.242 | 117.057 | 129.804 |
| 85 | 102.079 | 107.522 | 112.393 | 118.236 | 131.041 |
| 86 | 103.177 | 108.648 | 113.544 | 119.414 | 132.277 |
| 87 | 104.275 | 109.773 | 114.693 | 120.591 | 133.512 |
| 88 | 105.372 | 110.898 | 115.841 | 121.767 | 134.746 |
| 89 | 106.469 | 112.022 | 116.989 | 122.942 | 135.978 |
| 90 | 107.565 | 113.145 | 118.136 | 124.116 | 137.208 |
| 91 | 108.661 | 114.268 | 119.282 | 125.289 | 138.438 |
| 92 | 109.756 | 115.390 | 120.427 | 126.462 | 139.666 |
| 93 | 110.850 | 116.511 | 121.571 | 127.633 | 140.893 |
| 94 | 111.944 | 117.632 | 122.715 | 128.803 | 142.119 |
| 95 | 113.038 | 118.752 | 123.858 | 129.973 | 143.344 |
| 96 | 114.131 | 119.871 | 125.000 | 131.141 | 144.567 |
| 97 | 115.223 | 120.990 | 126.141 | 132.309 | 145.789 |
| 98 | 116.315 | 122.108 | 127.282 | 133.476 | 147.010 |
| 99 | 117.407 | 123.225 | 128.422 | 134.642 | 148.230 |
| 100 | 118.498 | 124.342 | 129.561 | 135.807 | 149.449 |
| 100 | 118.498 | 124.342 | 129.561 | 135.807 | 149.449 |

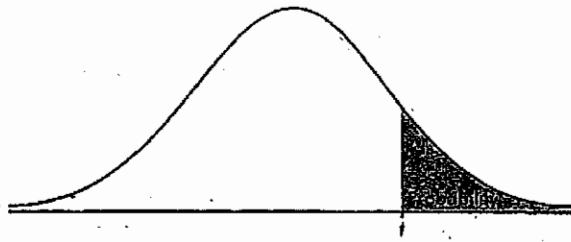


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

| df | Tail probability p | | | | | | | | | | | |
|----------|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | .679 | .849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | .679 | .848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | .678 | .846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | .677 | .845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | .675 | .842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| ∞ | .674 | .841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| | 50% | 60% | 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |
| | Confidence level C | | | | | | | | | | | |

F - Distribution ($\alpha = 0.05$ in the Right Tail)

| Denominator Degrees of Freedom df_2 | df_1 | Numerator Degrees of Freedom | | | | | | | | | |
|--|--------|------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| | | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | | 241.88 | 243.91 | 245.95 | 248.01 | 249.05 | 250.10 | 251.14 | 252.20 | 253.25 | 254.31 |
| 2 | | 19.396 | 19.413 | 19.429 | 19.446 | 19.454 | 19.462 | 19.471 | 19.479 | 19.487 | 19.496 |
| 3 | | 8.7855 | 8.7446 | 8.7029 | 8.6602 | 8.6385 | 8.6166 | 8.5944 | 8.5720 | 8.5494 | 8.5264 |
| 4 | | 5.9644 | 5.9117 | 5.8578 | 5.8025 | 5.7744 | 5.7459 | 5.7170 | 5.6877 | 5.6581 | 5.6281 |
| 5 | | 4.7351 | 4.6777 | 4.6188 | 4.5581 | 4.5272 | 4.4957 | 4.4638 | 4.4314 | 4.3985 | 4.3650 |
| 6 | | 4.0600 | 3.9999 | 3.9381 | 3.8742 | 3.8415 | 3.8082 | 3.7743 | 3.7398 | 3.7047 | 3.6689 |
| 7 | | 3.6365 | 3.5747 | 3.5107 | 3.4445 | 3.4105 | 3.3758 | 3.3404 | 3.3043 | 3.2674 | 3.2298 |
| 8 | | 3.3472 | 3.2839 | 3.2184 | 3.1503 | 3.1152 | 3.0794 | 3.0428 | 3.0053 | 2.9669 | 2.9276 |
| 9 | | 3.1373 | 3.0729 | 3.0061 | 2.9365 | 2.9005 | 2.8637 | 2.8259 | 2.7872 | 2.7475 | 2.7067 |
| 10 | | 2.9782 | 2.9130 | 2.8450 | 2.7740 | 2.7372 | 2.6996 | 2.6609 | 2.6211 | 2.5801 | 2.5379 |
| 11 | | 2.8536 | 2.7876 | 2.7186 | 2.6464 | 2.6090 | 2.5705 | 2.5309 | 2.4901 | 2.4480 | 2.4045 |
| 12 | | 2.7534 | 2.6866 | 2.6169 | 2.5436 | 2.5055 | 2.4663 | 2.4259 | 2.3842 | 2.3410 | 2.2962 |
| 13 | | 2.6710 | 2.6037 | 2.5331 | 2.4589 | 2.4202 | 2.3803 | 2.3392 | 2.2966 | 2.2524 | 2.2064 |
| 14 | | 2.6022 | 2.5342 | 2.4630 | 2.3879 | 2.3487 | 2.3082 | 2.2664 | 2.2229 | 2.1778 | 2.1307 |
| 15 | | 2.5437 | 2.4753 | 2.4034 | 2.3275 | 2.2878 | 2.2468 | 2.2043 | 2.1601 | 2.1141 | 2.0658 |
| 16 | | 2.4935 | 2.4247 | 2.3522 | 2.2756 | 2.2354 | 2.1938 | 2.1507 | 2.1058 | 2.0589 | 2.0096 |
| 17 | | 2.4499 | 2.3807 | 2.3077 | 2.2304 | 2.1898 | 2.1477 | 2.1040 | 2.0584 | 2.0107 | 1.9604 |
| 18 | | 2.4117 | 2.3421 | 2.2686 | 2.1906 | 2.1497 | 2.1071 | 2.0629 | 2.0166 | 1.9681 | 1.9168 |
| 19 | | 2.3779 | 2.3080 | 2.2341 | 2.1555 | 2.1141 | 2.0712 | 2.0264 | 1.9795 | 1.9302 | 1.8780 |
| 20 | | 2.3479 | 2.2776 | 2.2033 | 2.1242 | 2.0825 | 2.0391 | 1.9938 | 1.9464 | 1.8963 | 1.8432 |
| 21 | | 2.3210 | 2.2504 | 2.1757 | 2.0960 | 2.0540 | 2.0102 | 1.9645 | 1.9165 | 1.8657 | 1.8117 |
| 22 | | 2.2967 | 2.2258 | 2.1508 | 2.0707 | 2.0283 | 1.9842 | 1.9380 | 1.8894 | 1.8380 | 1.7831 |
| 23 | | 2.2747 | 2.2036 | 2.1282 | 2.0476 | 2.0050 | 1.9605 | 1.9139 | 1.8648 | 1.8128 | 1.7570 |
| 24 | | 2.2547 | 2.1834 | 2.1077 | 2.0267 | 1.9838 | 1.9390 | 1.8920 | 1.8424 | 1.7896 | 1.7330 |
| 25 | | 2.2365 | 2.1649 | 2.0889 | 2.0075 | 1.9643 | 1.9192 | 1.8718 | 1.8217 | 1.7684 | 1.7110 |
| 26 | | 2.2197 | 2.1479 | 2.0716 | 1.9898 | 1.9464 | 1.9010 | 1.8533 | 1.8027 | 1.7488 | 1.6906 |
| 27 | | 2.2043 | 2.1323 | 2.0558 | 1.9736 | 1.9299 | 1.8842 | 1.8361 | 1.7851 | 1.7306 | 1.6717 |
| 28 | | 2.1900 | 2.1179 | 2.0411 | 1.9586 | 1.9147 | 1.8687 | 1.8203 | 1.7689 | 1.7138 | 1.6541 |
| 29 | | 2.1768 | 2.1045 | 2.0275 | 1.9446 | 1.9005 | 1.8543 | 1.8055 | 1.7537 | 1.6981 | 1.6376 |
| 30 | | 2.1646 | 2.0921 | 2.0148 | 1.9317 | 1.8874 | 1.8409 | 1.7918 | 1.7396 | 1.6835 | 1.6223 |
| 40 | | 2.0772 | 2.0035 | 1.9245 | 1.8389 | 1.7929 | 1.7444 | 1.6928 | 1.6373 | 1.5766 | 1.5089 |
| 60 | | 1.9926 | 1.9174 | 1.8364 | 1.7480 | 1.7001 | 1.6491 | 1.5943 | 1.5343 | 1.4673 | 1.3893 |
| 120 | | 1.9105 | 1.8337 | 1.7505 | 1.6587 | 1.6084 | 1.5543 | 1.4952 | 1.4290 | 1.3519 | 1.2539 |
| ∞ | | 1.8307 | 1.7522 | 1.6664 | 1.5705 | 1.5173 | 1.4591 | 1.3940 | 1.3180 | 1.2214 | 1.0000 |

F - Distribution ($\alpha = 0.01$ in the Right Tail)

| Denominator Degrees of Freedom df_2 | df_1 | Numerator Degrees of Freedom | | | | | | | | | |
|--|--------|------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| | | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | | 6055.8 | 6106.3 | 6157.3 | 6208.7 | 6234.6 | 6260.6 | 6286.8 | 6313.0 | 6339.4 | 6365.9 |
| 2 | | 99.399 | 99.416 | 99.433 | 99.449 | 99.458 | 99.466 | 99.474 | 99.482 | 99.491 | 99.499 |
| 3 | | 27.229 | 27.052 | 26.872 | 26.690 | 26.598 | 26.505 | 26.411 | 26.316 | 26.221 | 26.125 |
| 4 | | 14.546 | 14.374 | 14.198 | 14.020 | 13.929 | 13.838 | 13.745 | 13.652 | 13.558 | 13.463 |
| 5 | | 10.051 | 9.8883 | 9.7222 | 9.5526 | 9.4665 | 9.3793 | 9.2912 | 9.2020 | 9.1118 | 9.0204 |
| 6 | | 7.8741 | 7.7183 | 7.5590 | 7.3958 | 7.3127 | 7.2285 | 7.1432 | 7.0567 | 6.9690 | 6.8800 |
| 7 | | 6.6201 | 6.4691 | 6.3143 | 6.1554 | 6.0743 | 5.9920 | 5.9084 | 5.8236 | 5.7373 | 5.6495 |
| 8 | | 5.8143 | 5.6667 | 5.5151 | 5.3591 | 5.2793 | 5.1981 | 5.1156 | 5.0316 | 4.9461 | 4.8588 |
| 9 | | 5.2565 | 5.1114 | 4.9621 | 4.8080 | 4.7290 | 4.6486 | 4.5666 | 4.4831 | 4.3978 | 4.3105 |
| 10 | | 4.8491 | 4.7059 | 4.5581 | 4.4054 | 4.3269 | 4.2469 | 4.1653 | 4.0819 | 3.9965 | 3.9090 |
| 11 | | 4.5393 | 4.3974 | 4.2509 | 4.0990 | 4.0209 | 3.9411 | 3.8596 | 3.7761 | 3.6904 | 3.6024 |
| 12 | | 4.2961 | 4.1553 | 4.0096 | 3.8584 | 3.7805 | 3.7008 | 3.6192 | 3.5355 | 3.4494 | 3.3608 |
| 13 | | 4.1003 | 3.9603 | 3.8154 | 3.6646 | 3.5868 | 3.5070 | 3.4253 | 3.3413 | 3.2548 | 3.1654 |
| 14 | | 3.9394 | 3.8001 | 3.6557 | 3.5052 | 3.4274 | 3.3476 | 3.2656 | 3.1813 | 3.0942 | 3.0040 |
| 15 | | 3.8049 | 3.6662 | 3.5222 | 3.3719 | 3.2940 | 3.2141 | 3.1319 | 3.0471 | 2.9595 | 2.8684 |
| 16 | | 3.6909 | 3.5527 | 3.4089 | 3.2587 | 3.1808 | 3.1007 | 3.0182 | 2.9330 | 2.8447 | 2.7528 |
| 17 | | 3.5931 | 3.4552 | 3.3117 | 3.1615 | 3.0835 | 3.0032 | 2.9205 | 2.8348 | 2.7459 | 2.6530 |
| 18 | | 3.5082 | 3.3706 | 3.2273 | 3.0771 | 2.9990 | 2.9185 | 2.8354 | 2.7493 | 2.6597 | 2.5660 |
| 19 | | 3.4338 | 3.2965 | 3.1533 | 3.0031 | 2.9249 | 2.8442 | 2.7608 | 2.6742 | 2.5839 | 2.4893 |
| 20 | | 3.3682 | 3.2311 | 3.0880 | 2.9377 | 2.8594 | 2.7785 | 2.6947 | 2.6077 | 2.5168 | 2.4212 |
| 21 | | 3.3098 | 3.1730 | 3.0300 | 2.8796 | 2.8010 | 2.7200 | 2.6359 | 2.5484 | 2.4568 | 2.3603 |
| 22 | | 3.2576 | 3.1209 | 2.9779 | 2.8274 | 2.7488 | 2.6675 | 2.5831 | 2.4951 | 2.4029 | 2.3055 |
| 23 | | 3.2106 | 3.0740 | 2.9311 | 2.7805 | 2.7017 | 2.6202 | 2.5355 | 2.4471 | 2.3542 | 2.2558 |
| 24 | | 3.1681 | 3.0316 | 2.8887 | 2.7380 | 2.6591 | 2.5773 | 2.4923 | 2.4035 | 2.3100 | 2.2107 |
| 25 | | 3.1294 | 2.9931 | 2.8502 | 2.6993 | 2.6203 | 2.5383 | 2.4530 | 2.3637 | 2.2696 | 2.1694 |
| 26 | | 3.0941 | 2.9578 | 2.8150 | 2.6640 | 2.5848 | 2.5026 | 2.4170 | 2.3273 | 2.2325 | 2.1315 |
| 27 | | 3.0618 | 2.9256 | 2.7827 | 2.6316 | 2.5522 | 2.4699 | 2.3840 | 2.2938 | 2.1985 | 2.0965 |
| 28 | | 3.0320 | 2.8959 | 2.7530 | 2.6017 | 2.5223 | 2.4397 | 2.3535 | 2.2629 | 2.1670 | 2.0642 |
| 29 | | 3.0045 | 2.8685 | 2.7256 | 2.5742 | 2.4946 | 2.4118 | 2.3253 | 2.2344 | 2.1379 | 2.0342 |
| 30 | | 2.9791 | 2.8431 | 2.7002 | 2.5487 | 2.4689 | 2.3860 | 2.2992 | 2.2079 | 2.1108 | 2.0062 |
| 40 | | 2.8005 | 2.6648 | 2.5216 | 2.3689 | 2.2880 | 2.2034 | 2.1142 | 2.0194 | 1.9172 | 1.8047 |
| 60 | | 2.6318 | 2.4961 | 2.3523 | 2.1978 | 2.1154 | 2.0285 | 1.9360 | 1.8363 | 1.7263 | 1.6006 |
| 120 | | 2.4721 | 2.3363 | 2.1915 | 2.0346 | 1.9500 | 1.8600 | 1.7628 | 1.6557 | 1.5330 | 1.3805 |
| ∞ | | 2.3209 | 2.1847 | 2.0385 | 1.8783 | 1.7908 | 1.6964 | 1.5923 | 1.4730 | 1.3246 | 1.0000 |