University of Swaziland

Final Examination – May 2018

BSc Env. Health I, BSc Comm. Health Nurs. I

Title of Paper : Calculus for Health Sciences
Course Number : EHS102
Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 2 questions in Section B.
4. Show all your working.
5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 Evaluate

a. $\lim_{x\to 3} \dfrac{3x - x^2}{x^2 - 9}$ [5 marks]

b. $\lim_{x\to \infty} \dfrac{3x - 1}{x^2 + 9}$ [5 marks]

A.2 a. Use the limit definition to find $f'(x)$ if

$$f(x) = 7x^2 - 2.$$ [10 marks]

b. Find y' if

i. $y = 4x^2 + 3 + 8\sqrt{x} - \dfrac{7}{x^2}$ [4 marks]

ii. $y = \pi^2 - 2e^{5x} - \cos 2x - \ln x^3$ [4 marks]

iii. $y = (2x + 1)e^{-2x}$ [5 marks]

iv. $y = \dfrac{x}{3 - 2x}$ [5 marks]

A.3 Integrate

a. $\displaystyle\int_1^9 \left(6x^2 - 6\sqrt{x} + 7\right)\mathrm{d}x$ [6 marks]

b. $\displaystyle\int_{\frac{1}{2}}^3 \left(10e^{2x} - \frac{3}{x} - \frac{5}{x^2}\right)\mathrm{d}x$ (correct to 2 d.p.) [6 marks]

Section B

Answer ANY 2 Questions in this section

B.1 a. Consider the function

$$y = (x-2)^6 + \ln(2x+1) + e^{-10x} - 60.$$

i. Find y' [3 marks]

ii. Find the equation of the tangent of y at $x = 0$. [5 marks]

b. A bullet is fired vertically upwards from the top of a 20m tower. If its height (in metres) is given by

$$h(t) = 20 + 245t - 4.9t^2,$$

where t is time in seconds after the shot, find

i. the maximum height reached by the bullet [4 marks]

ii. the total distance travelled by the bullet between $t = 4$ seconds and $t = 42$ seconds [5 marks]

iii. the speed at which it strikes the ground [8 marks]

B.2 a. Find the indicated derivative

i. $y = \cos 2x - \dfrac{7}{x^2}, \quad y'''$ [5 marks]

ii. $y = \ln\left(\dfrac{x}{1-2x}\right), \quad y'$ [5 marks]

b. Consider the function

$$f(x) = 10 + 54x - 2x^3.$$

i. Find the stationary points of $f(x)$ and determine the nature of each [9 marks]

ii. Find the inflexion point and y-intercept [2 marks]

iii. Make a sketch of the graph of $y = f(x)$. [4 marks]

B.3 a. Evaluate each integral using the specified method

i. $\displaystyle\int \frac{8x\,\mathrm{d}x}{\sqrt{x^2+5}}$, u-substitution [7 marks]

ii. $\displaystyle\int 4x^2e^{-2x}\,\mathrm{d}x$, tabular integration/integration by parts [8 marks]

b. Find the area of the region bounded by the parabola $y = x^2$ and the straight line $y = 2x + 8$. [10 marks]

B.4 a.

i. Resolve the rational expression

$$\frac{30x}{(x+2)(2x-1)}$$

into partial fractions. [10 marks]

ii. Hence, or otherwise, evaluate the integral

$$\int_2^{20} \frac{30x\,\mathrm{d}x}{(x+2)(2x-1)}.$$ [5 marks]

b. After the launch of a new product on 01 January 2018, the rate of sales (in thousands per month) is given by

$$S'(t) = 2 + 5e^{-0.1t},$$

where t is the number of months after 01 January 2018. Find

i. the total number of sales in the first year [5 marks]

ii. the total number of sales in the second year [5 marks]

END OF EXAMINATION