University of Swaziland

Final Examination – May/June 2019

BSc Env. Health I

Title of Paper : Calculus for Health Sciences

Course Number : EHS102

Time Allowed : Two (2) hours

Instructions:

1. This paper consists of 2 sections.

2. Answer ALL questions in Section A.

3. Answer ANY 2 questions in Section B.

4. Show all your working.

5. Begin each question on a new page.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A
Answer ALL Questions in this section

A.1 a. Evaluate

   i. $\displaystyle \lim_{x \to -2} \frac{x^3 - 8}{x^2 + 4}$ [3 marks]

   ii. $\displaystyle \lim_{x \to 0} \frac{x}{x^2 + \frac{1}{3}x}$ [3 marks]

   iii. $\displaystyle \lim_{x \to \infty} \frac{2 - 9x}{x^2 + 7}$ [3 marks]

   b. Use the limit definition to find $f'(x)$ if

$$f(x) = 7 - 3x^2.$$ [10 marks]

   c. Find y' if

   i. $y = 4x^3 - \dfrac{5}{x} - \dfrac{8}{\sqrt{x}} + \ln \sqrt{x}$ [6 marks]

   ii. $y = \dfrac{e^x}{e^x + e^{-x}}$ [7 marks]

   iii. $y = \left(1 + \sin 2x\right)^{10}$ [3 marks]

   d. Integrate

   i. $\displaystyle \int_1^{10} \left(4x + \frac{8}{\sqrt{x}} - \frac{3}{x}\right) dx$ (correct to 2 d.p.)   [7 marks]

   ii. $\displaystyle \int \left(4e^{-2x} + 6 \cos 0.2x\right) dx$ [4 marks]

   iii. $\displaystyle \int \left(\frac{2x - 5}{x}\right) dx$ [4 marks]

Section B

Answer ANY 2 Questions in this section

B.2 a. Evaluate

    i. $\displaystyle\lim_{x\to 0}\frac{x}{\sqrt{4+x}-2}$ [4 marks]

    ii. $\displaystyle\lim_{x\to 2}\frac{x+2}{(x-2)^2}$ [4 marks]

b. Use the *limit definition* to find $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ for $f(x)=-\dfrac{1}{x}$.   [10 marks]

c. Consider the function

$$y = 2e^{-3x} + \ln\left(x^2+1\right) + 8.$$

    Find the equation of the

    i. tangent at $x=0$ [5 marks]

    ii. normal at $x=0$ [2 marks]

B.3 a. Find the indicated derivative

    i. $y = x^6 - 32\sqrt{x}, \quad y^{iv}$ [4 marks]

    ii. $y = \ln\left(\dfrac{1+2x}{1+x}\right), \quad y'$ [4 marks]

    iii. $y = \dfrac{\sin x}{1+\cos x}, \quad y'$ [4 marks]

b. Consider the function

$$f(x) = 2x^3 - 3x^2 - 72x + 10.$$

    i. Find the stationary points of $f(x)$ and determine the nature of each

                                                       [7 marks]

    ii. Find the inflexion point and y-intercept   [3 marks]

    iii. Make a sketch of the graph of $y = f(x)$.   [3 marks]

B.4 a. Evaluate each integral using the specified method

  i. $\int 12x\sqrt{x^2+3}\,dx$ u-substitution   [7 marks]

  ii. $\int 9x^2 \sin 2x\,dx$,   tabular integration/integration by parts   [8 marks]

  b. Find the area of the region bounded by the parabola $y = x^2$ and the straight line $y = 2x + 8$.   [10 marks]

B.4 a.

  i. Resolve the rational expression

$$\frac{10x}{(x-2)(x+3)}$$

  into partial fractions.   [10 marks]

  ii. Hence, or otherwise, evaluate the integral

$$\int_3^{20} \frac{10x\,dx}{(x-2)(x+3)}.$$ [5 marks]

  b. After the launch of a new product on 01 January 2018, the rate of sales (in thousands per month) is given by

$$S'(t) = \frac{12}{(1+2t)^{\frac{3}{4}}},$$

  where t is the number of months after 01 January 2018. Find

  i. the total number of sales in the first year   [5 marks]

  ii. the total number of sales in the second year   [5 marks]

_____END OF EXAMINATION_____