15

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2005

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the seven (7) questions.

This examination paper should not be opened until permission has been granted by the invigilator.

15

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2005

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the seven (7) questions.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

(a) Consider the equivalence:

$$(P \vee Q) \wedge (\neg P \vee R) \equiv Q \vee R$$

(i) Prove the correctness of the equivalence by truth table. [7]

(ii) State the dual of the equivalence. [3]

(b) Prove by truth table that the following proposition is a contradiction:

$$\neg A \wedge C \wedge (A \Leftrightarrow B) \wedge (C \Rightarrow B)$$

[10]

Question 2

(a) State the contra-positive law of equivalence. [2]

(b) Prove the correctness of the contra-positive law using other laws of equivalence. [4]

(c) Simplify the following propositions using laws of equivalence:

(i) $\neg(A \wedge B \vee \neg(\neg A \Rightarrow B))$ [6]

(ii) $A \wedge B \wedge (C \vee \neg D) \vee A \wedge (B \vee D)$ [8]

Question 3

Verify the following by natural deduction:

(a) $A \wedge (B \Rightarrow C)$
$\underline{A \Rightarrow C}$
$\neg C \Rightarrow \neg D$

[4]

(b) $\neg(A \wedge B) \wedge C$
$\underline{B \wedge C \Leftrightarrow A}$
$\neg B$

[8]

(c) $\neg A \Rightarrow B \wedge C \vee D$
$\underline{\neg C \wedge \neg D}$
A

[8]

Question 4

(a) Draw a truth table for the Boolean function that is defined as follows:

- Input: 4 variables representing the 4 binary digits of an integer. The integer will lie in the range between zero and 15, inclusive.
- Output: 1 if the 4 input binary digits represent an integer that is a multiple of 4 (i.e. zero, 4, 8 or 12.) Otherwise output zero.

[4]

(b) Express the function defined in part (a) in disjunctive normal form (DNF).

[4]

(c) Using a Karnaugh map, minimize the expression given in the answer to part (b).

[6]

(d) Draw a circuit, consisting of NOR gates alone, that implements the function defined in part (a).

[6]

Question 5

(a) Distinguish between *level-triggered* and *edge-triggered* sequential logic circuits.
[2]

(b) Draw a complete circuit diagram of a D-latch, showing all logic gates contained in the device.
[6]

(c) Describe how D-latches may be used to construct a D-flip-flop.
[6]

(d) Describe how the values stored within a D-flip-flop will vary in response to input from a clock.
[6]

Question 6

Design a logic circuit that will take input from a clock and produce output in the form of a repeating sequence of integers from 2 up to 17 as follows: 2, 3, 4, …, 15, 16, 17, 2, 3, 4, …, 15, 16, 17, 2, …

The outputs of the circuit must be labelled S0, S1, S2, S3 and S4, each corresponding to one of the 5 binary digits of the output integer, starting from the least significant (or rightmost) digit, S0, and progressing to the most significant (or leftmost) digit , S4.
[20]

Question 7

(a) Copy the following predicates and circle each occurrence of a *free* variable:

$$\exists x(P(x) \wedge Q(y)) \wedge \forall y(R(x) \wedge S(y))$$

$$\forall x(A(x, y) \wedge \exists y(B(x)) \wedge \forall z(C(y, z)))$$

[6]

(b) Rewrite the following predicate such that all variables are *existentially* quantified:

$$\neg\forall x(P(x)) \vee \neg\exists y(\forall z(Q(y, z)))$$

[6]

(c) Give a model of the first predicate that is not a model of the second predicate:

$$\forall x(\exists y(P(x) \Rightarrow Q(x, y) \wedge \neg P(y)))$$

$$\exists x(P(x) \Rightarrow \forall y(Q(x, y) \wedge \neg P(y)))$$

[8]