

**UNIVERSITY OF SWAZILAND
FINAL EXAMINATION, MAY 2006**

Title of Paper : THEORY OF COMPUTATION

Course number : CS211

Time allowed : Three (3) hours.

- Instructions : (1) Read all the questions in Section-A and Section-B before you start answering any question.
- (2) Answer all questions in Section-A. Answer **any two** questions of section-B. Maximum mark is 100.
- (3) Use correct notation and show all your work on the answer script.

This paper should not be opened until the invigilator has granted permission.

SECTION-A

QUESTION 1 (marks 6 + 6+ 12). The following languages are given on $\Sigma = \{0, 1\}$. Assume $w \in \{0, 1\}^+$.

- (i). $L1 = \{w, |w| = 2m, \text{ for } m = 1, 2, 3, \dots\}$
- (ii). $L2 = \{w, \text{ such that } 01 \text{ is always a substring of } w\}$
- (iii). $L3 = \{w, \text{ such that } |w| \bmod 3 = 0\}$

The following set of words is given -

$\{\lambda, 0, 1, 01, 001, 0100, 00101, 111101, 11111, 100000, 0011110, 010101\}$

- (a). From the above set, write all the words belonging to L1, all the words belonging to L2 and all the words belonging to L3.
- (b). Write regular expressions representing L1, L2 and L3.
- (c). Design three deterministic finite acceptors (**dfa's**) accepting L1, L2, and L3 respectively.

QUESTION 2 (marks 6 + 8 + 12). The following non deterministic finite acceptor (**nfa**) is given :

$M = (\{q_0, q_1, q_2\}, \{0, 1\}, q_0, \delta, \{q_2\})$, where the transitions are given as :

$$\delta(q_0, 0) = q_1 ; \delta(q_0, 1) = q_0 ; \delta(q_1, 1) = q_2 ;$$

$$\delta(q_1, 0) = q_0 ;$$

- (i). Draw the transition digraph and write the state transition table of the **nfa**.
- (ii). Write the corresponding right linear grammar generating $L(M)$.
- (iii). Convert the above **nfa** into an equivalent **dfa**.

SECTION-B

Note: Answer **any two** questions in this section.

QUESTION 3(a) (marks 10). The grammar $G = (\{S\}, \{a,b\}, S, P)$ is given. The set of productions P is given as

$$\{ S \rightarrow aS \mid aSbS \mid \lambda \}.$$

Using G , write left most and right most derivations for $w = aaab$. Also show that G is ambiguous by drawing parse trees for w . What the complexity of G ?

QUESTION 3(b) (marks 15). Find Context Free (CFG) grammars that generate the following languages -

(i). $L(G1) = \{a^n b^m \mid \text{where } n, m \geq 0, n \neq 2m\}$

(ii). $L(G2) = \{a^n b^k c^k d^n \mid \text{where } n, k \geq 0\}$

write left most derivations for -

$w_1 = aaaaaabb$ (using $G1$) and $w_2 = aaabbccddd$ (using $G2$).

Include production number of your grammar at each step of derivation.

QUESTION 4(a) (marks 15). Design a deterministic pushdown automaton (**dpda**) to recognize the language -

$$L = \{ w \in \{a, b\}^+ \mid n_a(w) = n_b(w) \}$$

$n_a(w)$ and $n_b(w)$ are count of a's and b's in w respectively. Describe the functionality of your **dpda**. Write instantaneous descriptions for $w = abaabbab$.

QUESTION 4(b) (marks 4+6). Transform the following Grammar-

$$G = (\{S, A, B\}, \{a,b\}, S, P)$$

where the set of productions P is

$$\left\{ \begin{array}{l} S \longrightarrow aABB \mid aAA \\ A \longrightarrow aBB \mid C \\ B \longrightarrow bBB \mid A \\ C \longrightarrow a \end{array} \right\}$$

into Greibach Normal Form and design the **npda** that accepts $L(G)$.

QUESTION 5 (marks 5+10 + 5 + 5). Write the functional steps of the design of a Turing machine to compute –

$$F(x) = x \text{ div } 2$$

Assume x to be a non zero positive integer in unary representation. Also write the design and instantaneous descriptions using the values of x as 1111 and 11111 (in unary representation) for your Turing Machine.

(End of Examination Paper)