UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2006

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the seven (7) questions.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

a) Write the dual of the following logical equivalence: [2]

$\neg(A \wedge \mathbf{T}) \equiv \neg A \vee \mathbf{F}$

b) Prove by truth table the validity of the contra-positive law of logical equivalence. [4]

c) Prove by truth table that the following propositions are consistent: [10]

- $P \vee Q$
- $\neg R \Rightarrow P$
- $Q \Rightarrow \neg P$

d) Prove by perfect induction that the following conclusion is entailed by the three premises given in part c) above. [4]

$\neg(P \Leftrightarrow Q)$

Question 2

a) Prove the following using the laws of logical equivalence: [11]

$P \wedge Q \Rightarrow (R \Leftrightarrow \neg P) \equiv \neg(P \wedge Q \wedge R)$

b) Simplify the following proposition using the laws of logical equivalence: [9]

$A \wedge B \wedge C \vee A \wedge B \wedge \neg C \vee (A \wedge B \vee C) \wedge (C \Rightarrow A \wedge B)$

Question 3

Prove, by natural deduction, the validity of conclusions a) and b) based on the following premises:

- $P \vee Q \Rightarrow R$
- $S \Rightarrow Q \wedge R$
- $\neg Q$

a) $P \Rightarrow R$

[7]

b) $Q \Leftrightarrow S$

[13]

Question 4

a) Define the function `f(a,b,c)` in conjunctive normal form:

[8]

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

b) Implement a circuit for the function `g(a,b,c)`, using NOR gates alone:

[6]

$g(a,b,c) = \overline{a}b + c$

c) Write the following numbers in 9-bit binary according to the twos-complement system. Show all steps in your working.

[6]

i. 300

ii. -237

Question 5

a) Minimize the function f(a,b,c,d) using a Karnaugh map:

[9]

$$f(a,b,c,d) = abcd + \bar{a}b\bar{c}.\bar{d} + \bar{a}.\bar{b}.\bar{d}$$

Assume that the following inputs are impossible:

$\bar{a}cd$, $\bar{a}bd$

b) Minimize the function g(a,b,c,d) using the Quine-McCluskey method:

[11]

$$g(a,b,c,d) =$$

$$abcd + a\bar{b}cd + ab\bar{c}d + abc\bar{d} + a\bar{b}c\bar{d} + \bar{a}bcd + \bar{a}b\bar{c}d$$

Question 6

a) Draw the action tables of the RS latch and the JK flip flip.

[4]

b) Draw a circuit diagram showing how the JK flip flop is constructed from RS latches.

[8]

c) Draw a circuit that inputs a 3-bit number, and then performs modulo addition of 5 to it, outputting the 3-bit sum (see table below). The circuit should contain a number of adders.

[8]

input	***output***
0	5
1	6
2	7
3	0
4	1
5	2
6	3
7	4

Question 7

a) Copy the following predicate and circle each occurrence of a free variable: [3]

$$\neg\exists x(P(x, y) \Rightarrow Q(x) \wedge \neg\forall y(Q(y)))$$

b) Rewrite the predicate in part a) above such that all variables are: [8]

i. Universally quantified.

ii. Existentially quantified.

c) Give a model of the first predicate that is also a model of the second predicate: [9]

- $\forall x(\forall y(P(x, y) \Rightarrow Q(x, y)))$
- $\forall x(\forall y(Q(x, y) \wedge Q(y, x) \Rightarrow \neg P(x, y)))$