UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2006

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the seven (7) questions.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

a) Write the dual of the following logical equivalence: [2]

$\mathbf{F} \vee (A \wedge B) \equiv (\mathbf{F} \vee A) \wedge (\mathbf{F} \vee B)$

b) Prove by truth table the validity of the implication-elimination law of logical equivalence. [4]

c) Prove by truth table that the following propositions are consistent: [10]

- $P \Leftrightarrow \neg R$
- $Q \vee \neg P$
- $R \Rightarrow \neg Q$

d) Prove by perfect induction that the following conclusion is entailed by the three premises given in part c) above. [4]

$P \Leftrightarrow Q$

Question 2

a) Prove the following using the laws of logical equivalence: [11]

$\neg(P \vee \neg Q \Rightarrow R) \equiv Q \vee R \Rightarrow P \wedge \neg R$

b) Simplify the following proposition using the laws of logical equivalence: [9]

$(\neg A \vee B) \wedge \neg(\neg(C \wedge \mathbf{T}) \wedge \neg(A \Rightarrow B))$

Question 3

Prove, by natural deduction, the validity of conclusions a) and b) based on the following premises:

- $P \Leftrightarrow R \wedge S$
- $\neg(Q \wedge R)$
- $S \Rightarrow \neg P$

a) $\neg Q \vee \neg R \vee \neg S$ [7]

b) $\neg P$ [13]

Question 4

a) Define the function `f(a,b,c)` in conjunctive normal form: [8]

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

b) Implement a circuit for the function `g(a,b,c)`, using NAND gates alone: [6]

$g(a,b,c) = a+\overline{b}c$

c) Write the following numbers in 9-bit binary according to the twos-complement system. Show all steps in your working. [6]

i. 450

ii. -58

Question 5

a) Minimize the function f(a,b,c,d) using a Karnaugh map:

[9]

$$f(a,b,c,d) = acd + a\bar{b}.\bar{c}d + \bar{a}.\bar{b}cd + \bar{a}bc\bar{d}$$

Assume that the following input is impossible:

$\bar{a}bd$

b) Minimize the function g(a,b,c,d) using the Quine-McCluskey method:

[11]

$$g(a,b,c,d) =$$

$$abcd + a\bar{b}cd + abc\bar{d} + \bar{a}.\bar{b}cd + \bar{a}.\bar{b}.\bar{c}d + \bar{a}.\bar{b}c\bar{d} + \bar{a}.\bar{b}.\bar{c}.\bar{d}$$

Question 6

a) Draw a circuit diagram of the full adder.

[3]

b) Draw a circuit diagram of the D-latch, showing all logic gates and their interconnections.

[6]

c) Draw a circuit diagram of a divide-by-8 counter that is constructed from a number of JK flip flops. Label the clock input *Ck*, and label the 3 outputs q_0, q_1 and q_2.

[5]

d) Draw a timing diagram of the circuit given in c) above. It should graph the values of *Ck*, q_0, q_1 and q_2 over a period of 5 clock cycles.

[6]

Question 7

a) Copy the following predicate and circle each occurrence of a bound variable: [3]

$\forall x(\forall y(\exists z(P(y, z) \wedge \neg Q(y)) \vee P(z, x)))$

b) Rewrite the predicate in part a) above such that all variables are: [8]

i. Universally quantified.

ii. Existentially quantified.

c) Give a model of the first predicate that is not a model of the second predicate: [9]

- $\exists x(P(x) \wedge \forall y(P(y) \Rightarrow Q(x, y)))$
- $\forall x(P(x) \wedge \exists y(P(y) \Rightarrow Q(x, y)))$