UNIVERSITY OF SWAZILAND

Faculty of Science

Department of Computer Science

FINAL EXAMINATION, DECEMBER 2006

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the seven (7) questions.
Calculators may NOT be used.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

a) Construct a complete truth table of the following proposition. Hence determine whether the proposition is tautologous, contradictory or contingent.

$$P \vee \neg Q \Rightarrow P \wedge R \Leftrightarrow \neg(Q \vee R)$$

[14]

b) By truth table show that the following logical equivalence is valid:

$$F \vee \neg P \equiv \neg P \wedge T$$

[4]

c) Write the dual of the logical equivalence given in part a).

[2]

Question 2

a) Prove the following using the laws of logical equivalence:

$$\neg((R \Rightarrow \neg P) \wedge \neg(Q \wedge R)) \equiv R \wedge (P \vee Q)$$

[8]

b) Simplify the following proposition using the laws of logical equivalence:

$$(T \Rightarrow P) \wedge \neg(Q \vee F) \wedge (Q \vee R) \Rightarrow P$$

[12]

Question 3

By natural deduction from the following premises:

- $R \Leftrightarrow \neg P$
- $P \Rightarrow \neg Q$
- $Q \vee R$

... prove the following conclusions:

a) $\neg R \Rightarrow P$

[3]

b) $P \Rightarrow R$

[8]

c) $\neg P \wedge R$

[9]

Question 4

a) Define the function `f(a,b,c)` in conjunctive normal form:

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

[8]

b) Implement a circuit for the function `g(a,b,c)` using NAND gates alone:

$$g(a,b,c) = a\overline{b}+b\overline{c}$$

[8]

c) Write the following numbers in 9-bit binary according to the twos-complement system:

i. 73

[1]

ii. 243

[1]

iii. -112

[2]

Question 5

a) Minimize the function f(a,b,c,d) using a Karnaugh map:

$$f(a,b,c,d) = ab\overline{c} + acd + \overline{a}.\overline{b}cd + \overline{a}.\overline{b}.\overline{c}.\overline{d}$$

Assume that the following inputs are impossible:

$bc\overline{d}$, $\overline{a}.\overline{b}d$

[9]

b) Minimize the function g(a,b,c,d) using the Quine-McCluskey method:

$$g(a,b,c,d) =$$

$$abcd + abc\overline{d} + ab\overline{c}d + ab\overline{c}.\overline{d} +$$

$$a\overline{b}.\overline{c}d + a\overline{b}.\overline{c}.\overline{d} + \overline{a}bcd + \overline{a}b\overline{c}d$$

[11]

Question 6

a) What are the main characteristics of a *synchronous, sequential* circuit?

[2]

b) Describe the various ways in which the JK flip-flop can respond to its inputs.

[4]

c) Draw a complete labelled circuit diagram of the JK flip-flop, showing all logic gates.

[14]

Question 7

a) Write a predicate logic sentence containing 2 bound variables, named x and y, and 2 free variables, named v and w.

[2]

b) Consider the following predicate logic model:

- Universe of interpretation: set of all positive integers.
- Predicates:
  - Less(x, y) ≡ x is less than y
  - Divisible(x, y) ≡ x is divisible by y
  - Product(x, y, z) ≡ x multiplied by y equals z
  - Prime(x) ≡ x is a prime number

Translate the following statements into predicates under the above model:

i. All numbers are divisible by themselves.

[1]

ii. All prime numbers greater than 2 are odd.

[3]

iii. It is never the case that the product of 2 primes is also a prime.

[5]

iv. All non-prime numbers are the product of 2 prime numbers.

[5]

c) Give a single model for both the following predicates:

- $\forall x(P(x) \wedge \exists y(Q(x, y)))$
- $\exists x(\forall y(Q(x, y)))$

[4]