UNIVERSITY OF SWAZILAND

Faculty of Science

Department of Computer Science

MAIN EXAMINATION 2007

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the seven (7) questions.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

a) With the aid of a complete truth table, determine whether or not the following propositions are consistent with each other:

- $\neg R$
- $P \vee Q$
- $R \Rightarrow \neg P$
- $\neg(Q \wedge R)$

[12]

b) By truth table, prove that the *overlap* law of logical equivalence is valid.

[4]

c) By truth table, prove that the following entailment is valid:

$$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \models (P \Rightarrow R)$$

[4]

Question 2

a) Prove the following using the laws of logical equivalence:

$$\neg(P \Rightarrow Q) \wedge (Q \Rightarrow P \wedge \neg Q) \vee R \equiv Q \Rightarrow R$$

[8]

b) Simplify the following proposition using the laws of logical equivalence:

$$S \Rightarrow ((P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg(R \Leftrightarrow R) \vee S))$$

[12]

Question 3

By natural deduction from the following premises:

- $P \wedge Q \Rightarrow R$
- $\neg(Q \wedge R)$
- $\neg(P \Leftrightarrow R)$

... prove the following conclusions:

a) $P \Rightarrow \neg Q$

[6]

b) $Q \Rightarrow \neg P \wedge R$

[7]

c) $\neg Q$

[7]

Question 4

a) Define the function `f(a,b,c)` in conjunctive normal form:

a	**b**	**c**	**f(a,b,c)**
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

[8]

b) Implement a circuit for the function `g(a,b,c)` using NOR gates alone:

$$g(a,b,c) = (a+\overline{b}) \cdot (\overline{b}+c)$$

[8]

c) Write the following numbers in 9-bit binary according to the twos-complement system:

i. 49

[1]

ii. 200

[1]

iii. -81

[2]

Question 5

a) Minimize the function f(a,b,c,d) using a Karnaugh map:

$$f(a,b,c,d) =$$

$$abc\overline{d} + a\overline{b}c\overline{d} + a\overline{b}.\overline{c}.\overline{d} + \overline{a}bc\overline{d} + \overline{a}bcd + \overline{a}.\overline{b}cd$$

Assume that the following input is impossible:

$$\overline{a}.\overline{b}.\overline{d}$$

[10]

b) Minimize the function f(a,b,c,d) using a Karnaugh map:

$$f(a,b,c,d) =$$

$$ab\overline{c}d + \overline{a}b\overline{c}.\overline{d} + \overline{a}.\overline{b}cd + \overline{a}.\overline{c}d$$

Assume that the following inputs are impossible:

$$ab\overline{d},\ \overline{a}.\overline{b}.\overline{d}$$

[10]

Question 6

a) Draw a complete labelled circuit diagram of the full-adder, showing all logic gates.

[8]

b) Describe the various ways in which the D flip-flop can respond to its inputs.

[2]

c) Draw a complete labelled circuit diagram of the D-latch, showing all logic gates.

[6]

d) Explain, with the aid of a circuit diagram, how a D flip-flop may be constructed using D-latches.

[4]

Question 7

a) Write a predicate logic sentence containing 2 bound variables, named x and y, and 2 free variables, named v and w.

[2]

b) Prove the following logical equivalence:

$$\neg\forall x(\exists y(\neg P(x, y)) \Rightarrow \forall x(Q(x)))$$
$$\equiv \exists x(\exists y(\neg P(x, y)) \land \exists x(\neg Q(x)))$$

[7]

c) Consider the following predicate logic model:

- Universe of interpretation: set of all Uniswa students and lecturers.
- Predicates:
  - Same(x, y) ≡ x and y are the same person
  - Lecturer(x) ≡ x is a lecturer
  - Student(x) ≡ x is a student
  - Teach(x, y) ≡ person x teaches a course that is followed by person y

Translate the following statements into predicates under the above model:

i. No one can be both a lecturer and a student.

[2]

ii. Each lecturer has at least 1 student.

[4]

iii. Each student has at least 2 different lecturers.

[5]