

**UNIVERSITY OF SWAZILAND
MAIN EXAMINATION (SEM-I) Dec, 2008**

Title of Paper : THEORY OF COMPUTATION

Course number: CS211

Time allowed : Three (3) hours.

Instructions : (1) Read all the questions in Section-A and Section-B before you start answering any question.

(2) Answer all questions in Section-A and **any two** questions of section-B. Maximum mark is 100.

(3) Use correct notation and show all your work on the script.

This paper should not be opened until the invigilator has granted permission.

SECTION-A (Maximum marks 50)

Note: Answer **all** questions in this section.

Q1 (marks 6 + 6 +12): The following languages are given on symbol set $\{0, 1\}$. Assume that $u, v, w \in \{0, 1\}^*$.

(i). $L1 = \{uwv, |u| = |v| = 2\}$

(ii). $L2 = \{0w1\} \cup \{1w0\} \cup \{0, 1\}$

(iii). $L3 = \{w, (|w| \bmod 4 = 0)\}$

The following set of words is given -

$\{\lambda, 0, 1, 01, 001, 0100, 00011, 1111101, 0001111, 00000011, 001111011, 010101\}$

(a). From the above set, write all the words belonging to L1, all the words belonging to L2 and all the words belonging to L3.

(b). Write three regular expressions representing L1, L2 and L3 respectively.

(c). Design three deterministic finite acceptors (**dfa's**) accepting L1, L2, and L3 respectively.

Q2 (marks 6 + 8 + 12). The following non deterministic finite acceptor (**nfa**) is given :

$M = (\{q0, q1, q2, q3\}, \{0, 1\}, q0, \delta, \{q2, q3\})$, where the transitions are given as :

$$\delta(q0, 0) = \{q1\} ;$$

$$\delta(q1, 1) = \{q0, q2\} ; \quad \delta(q1, 0) = \{q3\} ;$$

$$\delta(q2, 0) = \{q1, q3\} .$$

(a). Draw the transition digraph of the above **M**.

(b). Compute $\delta^*(q0, w)$, where $w = 0000, 0111, 1000$ and 1010 .

(c). Find an equivalent **dfa** of **M**.

SECTION-B (Maximum marks 50)

Note: Answer **any two** questions in this section.

Q3(a) (marks 4+4+2). A context free grammar G is given as, $G = (\{E\}, \{a, b, c, +, *, (,)\}, E, P)$, the set of productions P is given as

$$\{ E \rightarrow E + E \mid E * E \mid (E) \mid a \mid b \mid c \}.$$

Using G , write left most derivations for $w_1 = a + b * c$ and $w_2 = a * b + c$. Taking examples of both, w_1 and w_2 , show that G is ambiguous by drawing two distinct parse trees for w_1 and w_2 . What is the complexity of G ?

Q3(b) (marks 9+6). Assuming n, m and $k \geq 0$. Find Context Free Grammars (CFG) G_1 and G_2 that generate the following languages. -

(i). $L(G_1) = \{a^n b^k c^{3n}\}$

(ii). $L(G_2) = \{a^n b^m c^k, \text{ such that } (m > k) \}$.

Write left most derivations for $w_1 = bb$, $w_2 = aabbcccccc$, $w_3 = \lambda$ using G_1 and $w_4 = bbb$, $w_5 = aabbc$ and $w_6 = bbbc$ using G_2 .

Include production number at each step of your derivation.

Q4(a) (marks 15). Design a deterministic pushdown automaton (**dpda**) to recognize the language -

$$L = \{w \in a^n b^m, \text{ such that } n > m \text{ and } w \text{ always starts with an } a \}$$

Clearly describe the steps as to how your **dpda** accepts and rejects words of L . Write instantaneous descriptions for $w_1 = aaab$ and $w_2 = aaabb$.

Q4(b) (marks 3 + 7). Design a non deterministic pushdown automaton (**npda**) to recognize the language generated by the grammar G , where -

$$G = (\{S, Z, B\}, \{a, b\}, S, P).$$

The set of productions P is

$$\begin{aligned} \{ S &\rightarrow aZ \mid aB \mid a \\ B &\rightarrow ZB \mid ZZB \mid b \\ Z &\rightarrow aZ \mid b \mid a \} \end{aligned}$$

Q5 (marks 15 + 5 + 5). Write the design of a Turing Machine (TM) to compute –

$$F(x) = x \bmod 2.$$

Assume x to be a non zero positive integer in unary representation. Clearly write the steps as to how your TM accomplishes the computations. Also write the instantaneous descriptions using the value of x as 111 and 111111 for your Turing Machine.

(End of Examination Paper)