UNIVERSITY OF SWAZILAND

Faculty of Science

Department of Computer Science

MAIN EXAMINATION 2008

Title of paper: INTRODUCTION TO LOGIC

Course number: CS235

Time allowed: Three (3) hours

Instructions: Answer any five (5) of the six (6) questions.

Special requirements: The use of electronic calculators is forbidden.

This examination paper should not be opened until permission has been granted by the invigilator.

Question 1

a) With the aid of a complete truth table, determine whether or not the following propositions are consistent with each other:

- R
- $\neg P$
- $P \Rightarrow Q$
- $\neg(P \vee Q) \vee R$

[12]

b) By truth table, prove that *De Morgan's* law of logical equivalence is valid.

[4]

c) By truth table, prove that the following entailment is valid:

$$P \wedge (P \Leftrightarrow Q) \models Q$$

[4]

Question 2

a) Prove the following using the laws of logical equivalence:

$$\neg(\neg(P \vee R \vee Q \wedge (P \vee R)) \vee \neg(P \Rightarrow \neg R)) \equiv \neg P \Leftrightarrow R$$

[9]

b) Simplify the following proposition as much as possible using the laws of logical equivalence:

$$(P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg(P \wedge Q) \Rightarrow R) \wedge (\neg(Q \wedge R) \Rightarrow \neg P)$$

[11]

Question 3

By natural deduction from the following premises:

- $P \vee R$
- $R \Leftrightarrow \neg Q \wedge P$
- $P \wedge Q \Rightarrow \neg R$

... prove the following conclusions:

a) P

[8]

b) $R \wedge \neg Q$

[8]

c) $\neg P \Rightarrow S$

[4]

Question 4

a) Define the function `f(a,b,c)` in disjunctive normal form:

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[7]

b) Implement a circuit for the function `g(a,b,c)` using NOR gates alone:

$$g(a,b,c,d) = a + \overline{b} + c\overline{d}$$

[8]

c) Write the following numbers in 10-bit binary according to the twos-complement system:

i. 437

[2]

ii. -180

[3]

Question 5

a) Minimize the function $f(a,b,c,d)$ using a Karnaugh map:

$$f(a,b,c,d) =$$

$$ab\bar{c}d + \bar{a}b\bar{c}.\bar{d} + \bar{a}.\bar{b}cd + \bar{a}.\bar{c}d$$

Assume that the following inputs are impossible:

$$ab\bar{d},\ \bar{a}.\bar{b}.\bar{d}$$

[9]

b) Minimize the function $g(a,b,c,d)$ using the Quine-McCluskey method:

$$g(a,b,c,d) =$$

$$abcd + \bar{a}bcd + \bar{a}.\bar{b}cd + \bar{a}bc\bar{d} +$$

$$\bar{a}b\bar{c}.\bar{d} + \bar{a}.\bar{b}c\bar{d} + \bar{a}.\bar{b}.\bar{c}.\bar{d}$$

[11]

Question 6

a) Distinguish between synchronous and asynchronous circuits.

[2]

b) Draw complete circuit diagram of the following devices, showing all logic gates:

i. RS latch

[5]

ii. Full adder

[8]

c) Explain the 2 main advantages of the D latch over the RS latch.

[5]