UNIVERSITY OF SWAZILAND

Faculty of Science

Department of Computer Science

MAIN EXAMINATION December 2010

Title of Paper: LOGIC FOR COMPUTER SCIENCE

Course Number: CS235

Time Allowed: 3 hours Total Marks: 100

Instructions to candidates:

This question paper consists of ***SIX (6)*** *questions. Answer any* ***FOUR (4)*** *questions.*
Marks are indicated in square brackets.
All questions carry equal marks.

SPECIAL REQUIREMENTS:

NO CALCULATORS ARE ALLOWED FOR THIS EXAM

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

QUESTION 1

a) i) State 2 limitations of propositional logic and 2 limitations of truth tables. [4]

ii) Explain the differences between propositional logic syntax and propositional logic semantics. [4]

iii) What is the difference between Entailment and Inference? [2]

iv) List 5 areas of application of Logic in Computer Science. [4]

b) Suppose you encounter three members A, B and C of the island of TuFa (remember that the Tu's always tell the truth, the Fa's always lie). They each give you a statement which we will assume you have translated into propositional logic as follows, where A denotes the statement: [8]
Member A says: $\neg (A \vee B \vee C) \wedge (\neg A \vee \neg B \vee \neg C)$

Use the truth table to determine whether A's proposition is a Tautology, a Contradiction or Contingent. To which tribe does this member belong?

c) Using identities, rewrite the proposition $(A \Rightarrow B \vee C) \wedge \neg B$ to one with fewer connectives. [3]

QUESTION 2

a) i) Using truth tables, show that $(A \vee C) \wedge (B \Rightarrow C) \wedge (C \Rightarrow A)$ is equivalent to $(B \Rightarrow C) \wedge A$ but not equivalent to $(A \vee C) \wedge (B \Rightarrow C)$. [8]

ii) From the truth table of i) above, determine the Conjunctive Normal Form (CNF) of $(B \Rightarrow C) \wedge A$ [4]

b) Using laws of equivalence to prove the following:
$A \Rightarrow (B \wedge C) \cong (A \Rightarrow B) \wedge (A \Rightarrow C)$ [4]

c) Use laws of equivalence to check if the proposition $(P \Rightarrow \neg Q) \vee (\neg R \Rightarrow P)$ is a tautology, a contradiction or neither. [4]

d) Given $(A \wedge (\neg B \vee C))$, $\neg C$ and $(\neg D \Rightarrow B)$ prove/ deduce $(D \wedge A)$ [5]

QUESTION 3

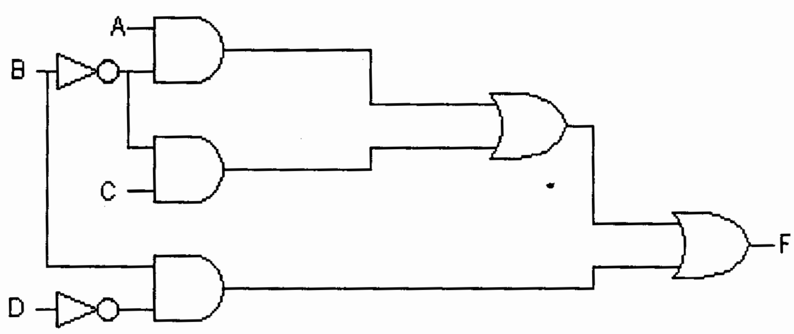
a) Digital circuits can be classified as either combination circuits or sequential circuits. Explain the differences between these circuits? Use diagrams in your explanation. [4]

b) A device accepts natural numbers in the range 0000 to 1111 that represent 0 to 15. The output F of the circuit is true if the input to the circuit represents a prime number and is false otherwise. [12]

  i) Draw the truth table for this function.
  ii) Hence, determine the canonical Sum of Products (SOP) and canonical Product of Sums (POS) expressions for the output F.
  iii) Write the short hand notation of the SOP and POS expressions.
  iv) Design a circuit using AND, OR and NOT gates to carry out this function.

c) Convert the following into SOP form and minimize using the Karnaugh map method.
$F = (AB + C)(B + \bar{C}\,D)$ [6]

d) Write down and simplify the logic function represented by the circuit diagram below: [3]



QUESTION 4

a) i) Briefly explain the difference between the Karnaugh map method and the Quine-McCluskey method. [3]

ii) Minimize the function $F(A,B,C,D) = \sum(0,1,2,3,6,7,8,9,14,15)$ using the Quine-McCluskey method. [10]

b) Flip flops can be implemented using R-S, D-type or J-K. Explain the different behaviors of these flip-flops. What additional logic is required to convert a J-K flip-flop into a D-type flip flop? [8]

c) Simplify the following Boolean expressions using Boolean theorems. [4]

$$\overline{(A + B)\overline{CD} + \overline{F}}$$

QUESTION 5

a) The state of a CPU register's contents is 100111.10101. What are its contents if it represents a positive real number? Show all your working. [5]

b) Find the fixed-point representation of the decimal number 47.125 Show all your working. [4]

c) Explain why the method of 2's complement arithmetic is commonly used as compared to other methods. [3]

d) State 2 examples of situations where fixed-point number representation is useful and often used. [2]

e) State 4 reasons for the wide spread adoption of digital technology and systems.[4]

f) With the aid of well-labeled circuit diagrams, distinguish between the Half adder and full adder circuits in the way they operate. [7]

QUESTION 6

a) Perform the following conversions: (Show ALL working).

i. Write down the 2's compliment representation of -123. [5]

ii. If 1010111 is the BCD representation of a decimal number, Find the decimal number. [5]

iii. Write down the Hexadecimal representation of 93. [4]

b) "If the program is running then there is at least 250K of RAM." Which of the following are equivalent to this statement? [5]

i) If there is at least 250K of RAM then the program is running.
ii) If there is less than 250K of RAM then the program is not running.
iii) The program will run only if there is at least 250K of RAM.
iv) If the program is not running then there is less than 250K of RAM.
v) A necessary condition for the program to run is that there are at least 250K of RAM.

c) Devise a truth table for a two input (A and B) logic system whose output is B only if A is zero; otherwise F inverts B. State what type of logic gate would be needed to implement such a system. [6]

<< End of Question Paper >>

Logical Equivalences Laws

Law(s)		Name
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$		Equivalence law
$p \rightarrow q \equiv \neg p \vee q$		Implication law
$\neg\neg p \equiv p$		Double negation law
$p \wedge p \equiv p$	$p \vee p \equiv p$	Idempotent laws
$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$	Commutative laws
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	de Morgan's laws
$p \wedge \mathrm{T} \equiv p$	$p \vee \mathrm{F} \equiv p$	Identity laws
$p \wedge \mathrm{F} \equiv \mathrm{F}$	$p \vee \mathrm{T} \equiv \mathrm{T}$	Annihilation laws
$p \wedge \neg p \equiv \mathrm{F}$	$p \vee \neg p \equiv \mathrm{T}$	Inverse laws
$p \wedge (p \vee q) \equiv p$	$p \vee (p \wedge q) \equiv p$	Absorption laws

Inference Rules

RULE Name	PREMISE	CONCLUSION (We can derive)
Modus Ponens **(mp)**	$A, A \rightarrow B$	B
Modus Tollens **(mt)**	$\neg B, A \rightarrow B$	$\neg A$
And Introduction **(con)**	A, B	$A \wedge B$
And Elimination **(simp)**	$A \wedge B$	A
And Elimination **(simp)**	$A \wedge B$	B
Disjunction Introduction **(add)**	A	$A \vee B$
Disjunction Introduction **(add)**	B	$A \vee B$
Double Negation **(dn)**	$\neg\neg A$	A
Unit Resolution **(ur)**	$A \vee B, \neg B$	A
Resolution (res)	$\mathbf{A \vee B, \neg B \vee C}$	$\mathbf{A \vee C}$