

**UNIVERSITY OF SWAZILAND
FINAL EXAMINATION DEC, 2013 (SEM-I)**

Title of Paper : THEORY OF COMPUTATION

Course number: CS211

Time allowed : Three (3) hours.

- Instructions : (1) Read this exam paper from page 1 to 4 completely,
before you start answering any question.
- (2) Answer all questions in Section-A and **any two**
questions from section-B. Maximum mark is 100.
- (3) Use correct notations and show all your work on the script.

This paper should not be opened until the invigilator has granted permission.

SECTION-A (Maximum marks 50)

Q1(a) (marks 6 + 6 +12). The following languages are given on symbol set $\{a, b\}$. Assume that $u, v, v_1, v_2, w \in \{a, b\}^*$.

(i). $L_1 = \{u v; (|u| > 3) \text{ and (symbol } b \text{ is not in } u)\}$

(ii). $L_2 = \{a v_1 a\} \cup \{b v_2 b\}$, $|v_1|$ and $|v_2| \geq 1$

(iii). $L_3 = \{w ; (|w| \bmod 3) \neq 0\}$

The following set of words is given -

$\{\lambda, a, b, ab, aab, abaa, aaabb, bbbbab, aaabbbb, aaaaaabb, aabbbbabb, ababab\}$

(a). From the above set, write all the words belonging to L_1, L_2 and L_3 .

(b). Write regular expressions representing L_1, L_2 and L_3 .

(c). Design three deterministic finite acceptors (**dfa's**) accepting L_1, L_2 , and L_3 respectively.

Q2 (marks 6 + 6 + 14). The following Machine is given:

$M = (\{q_0, q_1, q_2\}, \{0, 1\}, q_0, \delta, \{q_1\})$, where the transitions are given as :

$$\delta(q_0, 0) = q_1 ; \quad \delta(q_0, \lambda) = q_1 ;$$

$$\delta(q_1, 0) = \{q_0, q_2\} ; \quad \delta(q_1, 1) = q_1 ;$$

$$\delta(q_2, 0) = q_2 ; \quad \delta(q_2, 1) = q_1 .$$

(a). Draw the transition diagram of M . Write all the reasons, why M is an **nfa**.

(b). Compute $\delta^*(q_0, w)$ where $w = 000, 011$ and 010 .

(c). Convert the above M into an equivalent **dfa** and write its state transition table (STT).

SECTION-B (Maximum marks 50)

Note: Answer **any two** questions in this section.

Q3(a) (marks 10). Explain with examples the language of signed and unsigned integers. Write corresponding right linear grammar showing all your work. Give left most derivation trees of two integers, 23412 and -7642012.

Q3(b) (marks 15). Assuming n, m and $k \geq 0$. Find Context Free Grammars (CFG) G_1 and G_2 that generate the following languages. -

$$(i). L(G_1) = \{a^{2n} b^{m+n} c^m\}$$

$$(ii). L(G_2) = \{a^n b^m c^m d^n\}.$$

Write left most derivations for $w_1 = bbcc$, $w_2 = aaaabb$ and $w_3 = aabbbcc$ using G_1 and $w_4 = abcd$, $w_5 = aadd$, $w_6 = aabbccdd$ using G_2 .

Include production number at each step of derivation.

Q4(a) (marks 6 + 4 + 5). Design deterministic pushdown automaton (**dpda**) to recognize the language –

$$L = \{a^n b^{n+2}; n \geq 1\}$$

Describe the functional steps of your **dpda**. Write instantaneous descriptions for $w = abbb$ and $aabbb$.

Q4(b) (marks 6 + 4). Design a nondeterministic pushdown automaton (**npda**) to recognize the language generated by the following grammar –

$$G = (\{S, A, B, C\}, \{a, b, c\}, S, P)$$

where the set of productions P is

$$\left\{ \begin{array}{l} S \longrightarrow aaA \mid AC \\ A \longrightarrow aB \mid BC \mid a \\ B \longrightarrow bB \mid b \\ C \longrightarrow c \mid cC \end{array} \right\}$$

Convert G into Griebach Normal Form. Write instantaneous descriptions for $w = aabbbcc$ using your **npda**.

Q5 (marks 15 + 5 + 5). Write detailed description of the functional steps of the design of a Turing machine to compute –

$$F(x) = 2x$$

Assume x to be a non zero positive integer in unary representation. Also write the instantaneous descriptions using the value of x as 11 and 11111 (in unary representation) using your design.

(End of Examination Paper)