UNIVERSITY OF SWAZILAND
SUPPLEMENTARY EXAMINATION, JULY 2014

Title of Paper : THEORY OF COMPUTATION

Course number : CS 211

Time allowed : Three (3) hours.

Instructions : (1) Read all the questions from pages 1 to 4, before you start answering any question.

(2) Answer all questions in Section-A. Answer **any two** questions of Section-B. Maximum mark is 100.

(3) Use correct notations and show all your work on the answer script.

This paper should not be opened until the invigilator has granted permission.

SECTION-A (Maximum marks 50)

Q1 (marks 6 + 6+ 12). The following languages are given on symbol set $\{a, b\}$. Assume that – $u, v, w \in \{a, b\}^+$ and $\lambda \notin L1, L2$ or $L3$.

(i). $L1 = \{a\ w\ b\} \cup \{b\ w\ a\}$.

(ii). $L2 = \{u\ w\ v,\ |v| = |u| = 2\}$

(iii). $L3 = \{w,\ |w| \bmod 4 = 0\}$

The following set of words is given -

$\{\lambda$, a, b, ab, aab, abaa, aaabb, bbbbbab, aaabbbb, aaaaaabb, aabbbbabb, ababab$\}$

(a). From the above set, write all the words belonging to L1, all the words belonging to L2 and all the words belonging to L3.

(b). Write regular expressions representing L1, L2 and L3.

(c). Design three deterministic finite acceptors **(dfa's)** accepting L1, L2, and L3 respectively.

Q2 (marks 6 + 6 + 14) The following automaton is given :

$M = (\{q0, q1, q2\}, \{a, b\}, q0, \delta, \{q1\})$, where δ is given as :

$\delta(q0, a) = \{q1, q2\}$; $\delta(q0, b) = q0$;

$\delta(q1, a) = \{q0, q2\}$; $\delta(q1, b) = q1$;

$\delta(q2, a) = \{q0, q1\}$; $\delta(q2, b) = q2$

(a). Draw the transition digraph of M. Is M is an **nfa** ? Write all the reasons to your answer.

(b). Using M, compute $\delta^*(q0, w)$ completely, where w = aaa, aab and abb.

(c). Convert the above **nfa** into an equivalent **dfa**.

SECTION-B (Maximum marks 50)

Note: Answer **any two** questions in this section.

Q3(a) (marks 5 + 5). A Context Free Grammar (CFG) for simple arithmetic expressions is given as –

G = ({E, T, I}, {a, b, c, +, *, (,)},E, P)

where the set of productions P is

$$
\{ \quad
\begin{aligned}
E &\longrightarrow T \mid E + T, \\
T &\longrightarrow I \mid (E) \mid T * I \mid T * (E), \\
I &\longrightarrow a \mid b \mid c
\end{aligned}
\quad \}
$$

Write left most derivation for the expression, a + (b * c). Include the rule number used at each derivation step. Draw the corresponding derivation tree.

Q3(b) (marks 5 + 5 + 5). Find Context Free Grammars **(CFG)** G1 and G2 that generate the following languages. Assume $n \geq 0$ and $m \geq 0$ -

(i). L(G1) = $\{a^n b^{2n} c^{3m}\}$
(ii). L(G2) = $\{a^n b^m c^m d^n\}$

Test your CFG 's by writing left most derivations for -

w1 = abb, w2 = ccc, w3 = aabbbbccc (using G1) and
w4 = abcd, w5 = aadd, w6 = bbcc (using G2).

Include production number at each step of derivation.

Q4(a) (marks 10 + 5). Design a deterministic pushdown automaton **(dpda)** to recognize the language –

L = { w, w = $a^{2n} b^{2n}$, n is greater than or equal to 1 }

Describe the functional steps of your **dpda**. Write instantaneous descriptions for w1 = aaaabbbb and w2 = aaabbb.

Q4(b) (marks 6 + 4). Design a nondeterministic pushdown automaton **(npda)** to recognize the language generated by the grammar in Greibach Normal Form–

G = ({S, A, B}, {a,b},S, P)

where the set of productions P is –

$$\{ \quad S \longrightarrow aABB \mid aAA$$

$$A \longrightarrow aBB \mid a$$

$$B \longrightarrow bBB \mid aBB \mid a \quad \}$$

Write instantaneous descriptions of your **npda** for w = aaabaaaa.

Q5 (marks 15 + 10). The following Turing Machine is given -

TM = ({q0, q1, q2, q3}, {1}, {1, □}, δ, q0, □, {q3})

δ(q0, 1) = {q0, x, R)
δ(q0, □) = {q1, □, L)
δ(q1, x) = {q2, 1, R)
δ(q2, 1) = {q2, 1, R)
δ(q2, □) = {q1, 1, L)
δ(q1, 1) = {q1, 1, L)
δ(q1, □) = {q3, □, R)

Analyze each transition and describe its functionality. Write the instantaneous descriptions when this TM starts in q0 at the left most symbol of input string - 111, i.e.

$q_0\ 111 \vdash^* \ ?$

(End of Examination Paper)