University of Swaziland
Department Of Computer Science
DECEMBER 2013

Title of paper: *Introduction to Logic*

Course number: *CS235*

Time Allowed: *Three (3) hours*

Instructions:

- *This paper consists of **six (6)** questions.*
- *Each question is worth 25 marks*
- *Answer **any four (4)** questions from questions 1 to 6.*

SPECIAL REQUIREMENT:

NO CALCULATORS ARE ALLOWED FOR THIS EXAMINATION PAPER.

This paper may not be opened until permission has been granted by the invigilator

QUESTION 1

(i) Define the following terms

- (a) Proposition [2]
- (b) Predicate [2]
- (c) Valid Argument [2]
- (d) Tautology [2]
- (e) Contingent [2]

(ii) Let P = "Sipho is healthy"

Q = "Sipho is wealthy"

R = "Sipho is wise"

Represent as propositional expressions the following statements:

- (a) Sipho is healthy and wealthy but not wise [2]
- (b) Sipho is not wealthy but he is healthy and wise [2]
- (c) Sipho is neither healthy nor wealthy nor wise [2]

(iii) Consider the sentence: **If the test is cancelled we shall have a party.**

- (a) Identify the atomic propositions in the sentence. [2]
- (b) Translate the sentence into a propositional form and write down its truth table. [2]

(iv) Define suitable predicates and then express the following statement as a logical expression:

All the boys failed the mathematics test. [5]

QUESTION 2

At University of Swaziland, students are registered in courses. At the end of the year, each course is allocated a mark, and a student is declared to have passed a course if the mark obtained in that course is greater than 50. A course can be supplemented if the mark is greater than 40 but less than 50.

Using prolog notation:

(i) Define suitable ground predicates to express the following facts in a knowledge base:

- Five (5) students: **gugu, kim, joe, musa, fana,** [3]
- Three (3) courses): **cs211, m220, b204** [2]
- Five (5) student registration details. Each registration specifies the student name, the course and the mark obtained. A student may register for more than one course. Two of the students should not be registered in any course. [4]

(ii) Define a rule predicate, called **pass (S,C),** that returns true if student S registered in course **C** and passed the course. [3]

(iii) Define a rule predicate, called **supplement (S,C),** that returns true if student **S** registered in course C, and can supplement the course. [3]

(iv) Write a query for each of the following:- in each case indicate the expected result of the query [based on your facts in (i)].

(a) Determine if **gugu** is a student. [1]

(b) Determine if **fana** is registered in B204. [1]

(c) Find all courses that have a mark less than 30 [2]

(d) Find all students who failed some course [3]

(e) Find all students who are not registered in any course [3]

QUESTION 3

(i) Convert the decimal number **154** to its binary equivalent. [2]

(ii) Convert the binary number **101110** to its hexadecimal equivalent. [3]

(iii)Consider a Boolean function that takes 4 inputs representing the 4 binary digits of any integer number between 0 and 15. The function return true if the value of the integer number is a multiple of 3 (i.e. 3, 6, 9, etc), and false otherwise.

(a) Draw a truth table for the boolean function. [4]

(b) Based on the truth table obtained in (a) above, write a logical expression of the function in:

- Conjunctive normal form. [3]
- Disjunctive normal form. [3]

(c) Use a Karnaugh map to obtain a minimized logical expression of the function. [5]

(d) Based on your answer in (c) above, draw a circuit diagram that implements the function defined in (a). [5]

QUESTION 4

(i) Using a truth table show that **$(A \vee C) \wedge (B \rightarrow C) \wedge (C \rightarrow A)$** is equivalent to **$(B \rightarrow C) \wedge A$** [6]

(ii) From the truth table in (i) above, determine the conjunctive normal form of the expression: **$(B \rightarrow C) \wedge A$.** [4]

(iii)Using the laws of equivalence, prove that **$A \rightarrow (B \wedge C)$** is equivalent to **$(A \rightarrow B) \wedge (A \rightarrow C)$**. Show all your workings. [5]

(iv) Given **$(A \wedge (\neg B \vee C))$**, **$(\neg D \rightarrow B)$** and **$\neg C$**, use rules of inference to prove/deduce **$(D \wedge A)$**. Show all your workings. [5]

(v) Given **$(P \vee R)$**, **$(R \leftrightarrow \neg Q \wedge P)$** and **$(P \wedge Q \rightarrow \neg R)$**, prove/deduce **$(R \wedge \neg Q)$**. Show all your workings. [5]

QUESTION 5

(i) Consider the following arguments:

- Alice is a Math major. Therefore, Alice is either a Math major or a History major.
- If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
- If it snows today, the University will close. The University is not closed today. Therefore, it did not snow today.
- If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

(a) For each argument above, describe suitable propositional variables and then express each argument as a sequent. [8]

(b) For each argument, state the rule of inference that is used to reach the conclusion? [4]

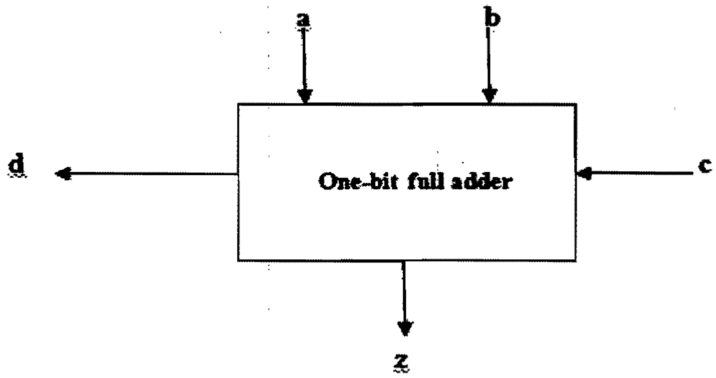
(ii) State the resolution rule of inference, and use a truth table to verify the validity of the conclusion. [5]

(iii) Use the resolution rule of inference to prove that given $(A \wedge B) \vee C$ and $C \rightarrow D$, we can conclude $A \vee D$. Show all your workings. [8]

QUESTION 6

(i) Using binary arithmetic evaluate the binary arithmetic expression: 101110+ 1101111 [4]

(ii) The binary ripple-carry addition algorithm used in (i), can be implemented using a one-bit full adder that takes two input bits **a** and **b**, and a carry-in bits **c** and compute a sum bit **z** and a carry-out bit **d.** The diagram below illustrates the input and outputs of a one-bit adder.



(a) Draw a truth table for the one-bit full-adder. [6]

(b) Based on the truth table obtained in (a) write the logical expressions for the sum bit **z** and the carry-out bit **d.** [4]

(c) Simplify the logical expressions obtained in (b), and draw one circuit diagram for the one-bit full adder. [6]

(iii) Using a diagram similar to one shown above for the one-bit adder, draw a diagram that illustrates how a 3-bit adder could be implemented using a combination of one-bit adders. [5]