University of Swaziland
Department Of Computer Science
DECEMBER 2014

Title of paper: *Introduction to Logic*

Course number: *CS235*

Time Allowed: *Three (3) hours*

Instructions:

- *This paper consists of* ***six (6)*** *questions.*
- *Each question is worth* ***25 marks***
- *Answer* ***any four (4)*** *questions from questions 1 to 6.*

SPECIAL REQUIREMENT:

NO CALCULATORS ARE ALLOWED FOR THIS EXAMINATION PAPER.

This paper may not be opened until permission has been granted by the invigilator

QUESTION 1

Miss Take takes prides in dealing appropriately with all weather conditions. She follows a strict set of rules that ensure that she never lives up to her name. She never goes to work if it is raining in November. If she does go to work, she takes an umbrella when it is raining and a coat when it is windy. She always wears a hat when she goes to work, unless it is windy. If it is windy in November she switches on her central heating system.

Based on this policy, we can conclude that Miss Take never gets wet when it is raining and never gets cold when it is windy.

(i) Identity and define suitable atomic propositional variables in the statement given above. [7]

(ii) Using the variables defined in (i) above, express the rules followed by Miss Take using propositional expressions. [12]

(iii) Using the rules of inference, determine the validity of the conclusion in the above paragraph. [6]

QUESTION 2

(i) Define the following terms

(a) Proposition [2]

(b) Predicate [2]

(c) Tautology [2]

(d) Valid argument [2]

(ii) Consider the sentence: **If we boycott the test and the lecturer does not give us a make up test, then we shall have all fail**

(a) Identify the atomic propositions in the sentence. [2]

(b) Translate the sentence into a propositional expression and write down its truth table. [3]

(iii) Translate the following into logic. [*All variables and domains must be adequately defined before they are used.*]

(a) Mr Magu is healthy and wealthy but not wise. [3]

(b) No school buses are purple. [3]

(c) If an integer is not Odd, then it is even. [3]

(d) The only even prime number is 2. [3]

QUESTION 3

At University of Swaziland, students are registered in courses. At the end of the year, each course is allocated a mark, and a student is declared to have passed a course if the mark obtained in that course is greater than or equal 50. A course can be supplemented if the mark is greater than 40 but less than 50.

Using prolog notation:

(i) Define suitable ground predicates to express the following facts in a knowledge base:

- Five (5) students: **gugu, kim, joe, musa, fana,** [3]
- Three (3) courses: **cs211, m220, b204** [2]
- Five (5) student registration details. Each registration specifies the student name, the course and the mark obtained. A student may register for more than one course. Two of the students should not be registered in any course. [4]

(ii) Define a rule predicate, called **pass (S,C),** that returns true if student **S** registered in course **C** and passed the course. [3]

(iii) Define a rule predicate, called **supplement (S,C),** that returns true if student **S** registered in course C, and can supplement the course. [3]

(iv) Write a query for each of the following:- in each case indicate the expected result of the query [based on your facts in (i)].

(a) Determine if **gugu** is a student. [1]

(b) Determine if **fana** is registered in B204. [1]

(c) Find all courses that have a mark less than 30 [2]

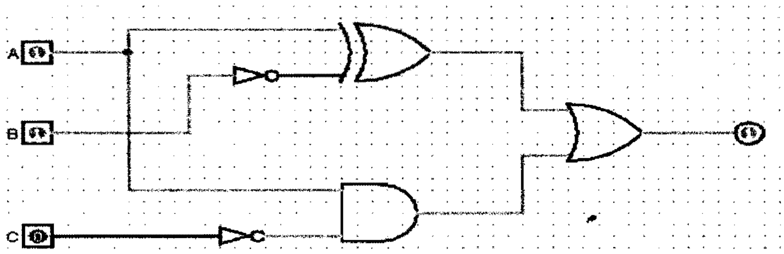
(d) Find all students who failed some course [3]

(e) Find all students who are not registered in any course [3]

QUESTION 4

1. Perform the following conversions and show all your workings. (*no marks for just writing the answer*)
   - (a) The Decimal number 345 to its binary equivalent. [2]
   - (b) The Decimal number 345 to its base 4 equivalent. [2]
   - (c) Convert the Octal number 3174 to its hexadecimal. [3]
   - (d) The binary number 110110011 to its base 3 equivalent [3]

2. Analyze the following circuit



   - (a) Draw the truth table. [5]
   - (b) Write the output expression using sum of products [5]
   - (c) Write the output expression using product of sums [5]

QUESTION 5

(i) Using the laws of equivalence, prove that **A→ (B∧C)** is equivalent to **(A→B)** ∧ **(A→C).** Show all your workings. [5]

(ii) Given **(A ∧ (¬B ∨C)), (¬D →B)** and **¬C** , use rules of inference to prove/deduce **(D ∧A)** . Show all your workings. [5]

(iii) Show that the following equivalence holds:

$$\neg \forall x P(x) \equiv \exists x \neg P(x) \qquad [5]$$

(iv) Using a counter-example, show that ∃ does not distribute over ∧. [5]

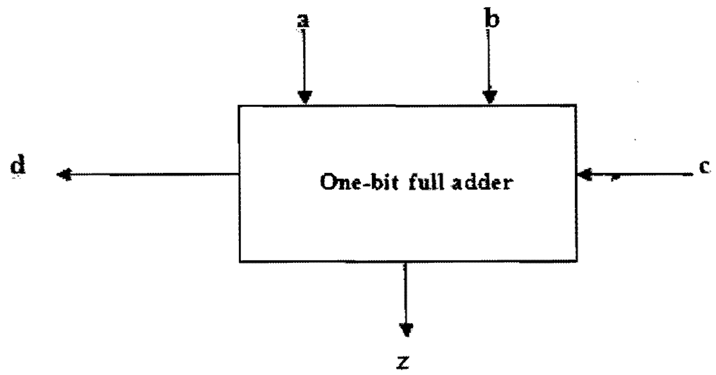
(v) Express the following argument using logical expression, and use rules of inference to prove the validity of the argument.

> **All politicians are crooks. Sipho is a politician. Therefore, we can conclude that Sipho is a crook.** [5]

QUESTION 6

(i) Using binary arithmetic evaluate the binary arithmetic expression: 101110+ 1101111 [4]

(ii) The binary ripple-carry addition algorithm used in (i), can be implemented using a one-bit full adder that takes two input bits **a** and **b**, and a carry-in bits **c** and computes a sum bit **z** and a carry-out bit **d.** The diagram below illustrates the input and outputs of a one-bit adder.



(a) Draw a truth table for the one-bit full-adder. [6]

(b) Based on the truth table obtained in (a) write the logical expressions for the sum bit **z** and the carry-out bit **d**. [4]

(c) Simplify the logical expressions obtained in (b), and draw a circuit diagram for the one-bit full adder. [6]

(iii) Using a diagram similar to the one shown above for the one-bit adder, draw a diagram that illustrates how a 3-bit adder could be implemented using a combination of one-bit adders. [5]