University of Swaziland

Department Of Computer Science
Supplementary Examination
July 2015

Title of paper: *Introduction to Logic*

Course number: *CS235*

Time Allowed: *Three (3) hours*

Instructions:

- *Answer any other four (4) questions from questions 1 to 5*
- *Each question carries 25 marks*

This paper may not be opened until permission has been granted by the invigilator

QUESTION 1

a) i) State 2 limitations of propositional logic and 2 limitations of truth tables. [4]

ii) Explain the differences between propositional logic syntax and propositional logic semantics. [4]

iii) What is the difference between a proposition and a predicate? [2]

iv) List 3 areas of application of Logic in Computer Science. [4]

b) Suppose you encounter three people, Musa, Sipho and Jane, of the island of TuFa (remember that the Tu's always tell the truth, the Fa's always lie). They each give you a statement which we will assume you have translated into propositional logic with variables A, B and C:
Musa says: $\neg (A \vee B \vee C) \wedge (\neg A \vee \neg B \vee \neg C)$

Use the truth table to determine whether Musa's proposition is a Tautology, a Contradiction or Contingent. To which tribe (Tu's or Fa's) does Musa belong? [8]

c) Using identities, rewrite the proposition $(A \Rightarrow B \vee C) \wedge \neg B$ to one with fewer connectives. [3]

QUESTION 2

a) i) Using truth tables, show that $(A \vee C) \wedge (B \Rightarrow C) \wedge (C \Rightarrow A)$ is equivalent to $(B \Rightarrow C) \wedge A$ but not equivalent to $(A \vee C) \wedge (B \Rightarrow C)$. [8]

ii) From the truth table of i) above, determine the Conjunctive Normal Form (CNF) of $(B \Rightarrow C) \wedge A$ [4]

b) Using laws of equivalence to prove the following:
$A \Rightarrow (B \wedge C) \cong (A \Rightarrow B) \wedge (A \Rightarrow C)$ [4]

c) Use laws of equivalence to check if the proposition $(P \Rightarrow \neg Q) \vee (\neg R \Rightarrow P)$ is a tautology, a contradiction or neither. [4]

d) Given $(A \wedge (\neg B \vee C))$, $\neg C$ and $(\neg D \Rightarrow B)$ prove/ deduce $(D \wedge A)$ [5]

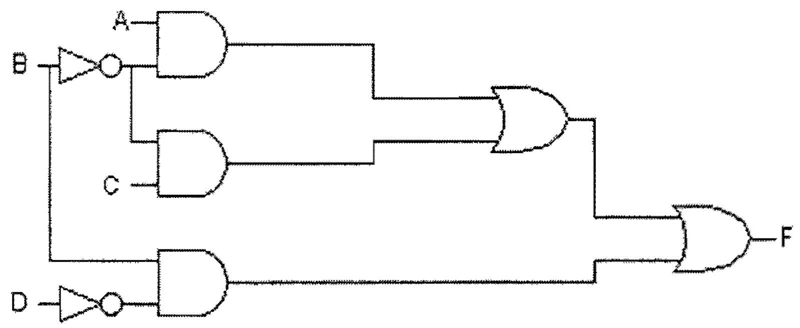
QUESTION 3

a) A device accepts natural numbers in the range 0000 to 1111 that represent 0 to 15. The output F of the circuit is true if the input to the circuit represents a prime number and is false otherwise. [14]

- i) Draw the truth table for this function.
- ii) Hence, determine the canonical Sum of Products (SOP) and canonical Product of Sums (POS) expressions for the output F.
- iii) Write the short hand notation of the SOP and POS expressions.
- iv) Design a circuit using AND, OR and NOT gates to carry out this function.

b) Convert the following into SOP form and minimize using the Karnaugh map method.
$\mathbf{F} = (AB + C)(B + \overline{C}\,D)$ [6]

c) Write down and simplify the logic function represented by the circuit diagram below: [5]



QUESTION 4

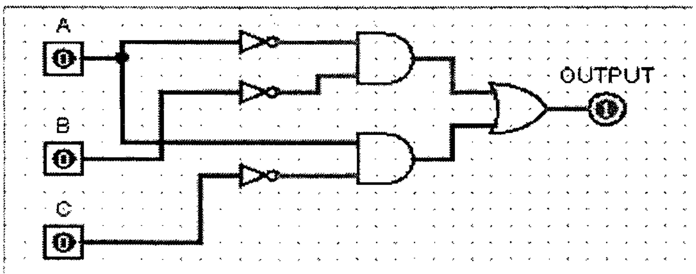
a) Perform the following conversions and show all your workings. (*no marks for just writing the answer*)

   i) The Decimal number 425 to its binary equivalent. [2]
   ii) The Decimal number 435 to its base 4 equivalent. [2]
   iii) Convert the Octal number 435 to its hexadecimal. [2]
   iv) The decimal number 11011001 to its base 3 equivalent [2]

b) Use the equivalence rules to simplify the following Boolean expression. (*Show all steps and state the rule being applied at each step.*) [4]

$$\overline{\left(A+\overline{B}\right)}+\overline{C}$$

c) Analyze the following circuit



   i) Draw the truth table. [5]
   ii) Write the output expression in Conjunctive Normal Form [4]
   iii) Write the output expression in Disjunctive Normal Form [4]

QUESTION 5

a) For each of the following statement, state whether the statement is true or false.

(i) If it is not possible for the conclusion of an argument to be false, then the argument is valid. [1]

(ii) If the conclusion of a valid argument is true, the premises must be true as well. [2]

b) Is the following argument valid? Show your working. [5]

"If Musa Maseko is unmarried, then Musa Maseko is a bachelor. Musa Maseko is a bachelor. Therefore, Musa Maseko is unmarried."

c) Is the following argument valid? Show your working. [5]

$A \vee B$
$A \rightarrow C$
$B \rightarrow C$

$\therefore C$

d) Symbolize the following argument . [5]

All Swazis are friendly. Joe is not friendly. Therefore, Joe is not a Swazi.

e) Consider the set of integer numbers, and let

N(x) : x is a non-negative integer
E(x) : x is even
O(x) : x is odd
P(x) : x is prime

Translate the following into logical notation

(i) There exists an even number [1]
(ii) 2 is the only prime number [1]
(iii)Some prime numbers are even [1]
(iv) If integer x is even, then 2x is even [1]

f) Is the expression $\exists x\ (P(x) \wedge Q(x))$ equivalent to $\exists x P(x) \wedge Q(x)$?. Explain your answer. [3]