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UNIVERSITY OF SWAZILAND

Faculty of Science

Department of Computer Science

MAIN EXAMINATION December 2015

Title of Paper: INTRODUCTION TO LOGIC

Course Number: CS235

Time Allowed: **3 hours** Total Marks: **100**

Instructions to candidates:

*This question paper consists of **SIX (6)** questions. Answer any **FOUR (4)** questions.*
Marks are indicated in square brackets.
All questions carry equal marks.

SPECIAL REQUIREMENTS:

NO CALCULATORS ALLOWED

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

QUESTION 1

a) i) State 2 limitations of propositional logic and 2 limitations of truth tables. [4]

ii) Explain the differences between propositional logic syntax and propositional logic semantics. [4]

iii) What is the difference between Entailment and Inference? [2]

iv) List 4 areas of application of Logic in Computer Science. [4]

b) Suppose you encounter three members A, B and C of the island of TuFa (remember that the Tu's always tell the truth, the Fa's always lie). They each give you a statement which we will assume you have translated into propositional logic as follows, where A denotes the statement: [8]

Member A says: $\neg$ **(A $\vee$ B $\vee$ C) $\wedge$ ($\neg$ A $\vee \neg$ B $\vee \neg$ C)**

Use the truth table to determine whether A's proposition is a Tautology, a Contradiction or Contingent. To which tribe does this member belong?

c) Using identities, rewrite the proposition **(A$\Rightarrow$ B $\vee$ C) $\wedge$ $\neg$ B** to one with fewer connectives. [3]

QUESTION 2

a) i) Using truth tables, show that **(A $\vee$ B) $\rightarrow$ C** is equivalent to **(A$\rightarrow$C) $\wedge$ (B $\rightarrow$C)** [5]

ii) From the truth table of i) above, determine the Conjunctive Normal Form (CNF) and the Disjunctive Normal Form (DNF) of **(A $\vee$ B) $\rightarrow$ C** [6]

b) Three boys, Melusi, Brian and Nkosi are caught, suspected of breaking the glass in a lab.

- Melusi says: "Brian did it; Nkosi is innocent".
- Brian says: "If Melusi is guilty then so is Nkosi".
- Nkosi says: "I didn't do it; one of the others did".

i) Are the statements consistent? [6]

ii) Assuming that everyone is innocent, who told lies? [2]

iii) Assuming that everyone's statement is true, who is innocent and who is guilty? [2]

c) Given **(A $\vee$ B) $\rightarrow \neg$ C, $\neg$C $\rightarrow$ D, A** prove or deduce **D** [4]

QUESTION 3

At University of Swaziland, students are registered in courses. At the end of the year, each course is allocated a mark, and a student is declared to have passed a course if the mark obtained in that course is greater than 50. A course can be supplemented if the mark is greater than 40 but less than 50.

Using prolog notation:

(i) Define suitable ground predicates to express the following facts in a knowledge base:

- Five (5) students: **gugu, kim, joe, musa, fana,** [3]
- Three (3) courses): **cs211, m220, b204** [2]
- Five (5) student registration details. Each registration specifies the student name, the course and the mark obtained. A student may register for more than one course. Two of the students should not be registered in any course. [4]

(ii) Define a rule predicate, called **pass (S,C),** that returns true if student S registered in course **C** and passed the course. [3]

(iii) Define a rule predicate, called **supplement (S,C),** that returns true if student **S** registered in course C, and can supplement the course. [3]

(iv) Write a query for each of the following:- in each case indicate the expected result of the query [based on your facts in (i)].

(a) Determine if **gugu** is a student. [1]

(b) Determine if **fana** is registered in b204. [1]

(c) Find all courses that have a mark less than 30 [2]

(d) Find all students who failed some course [3]

(e) Find all students who are not registered in any course [3]

QUESTION 4

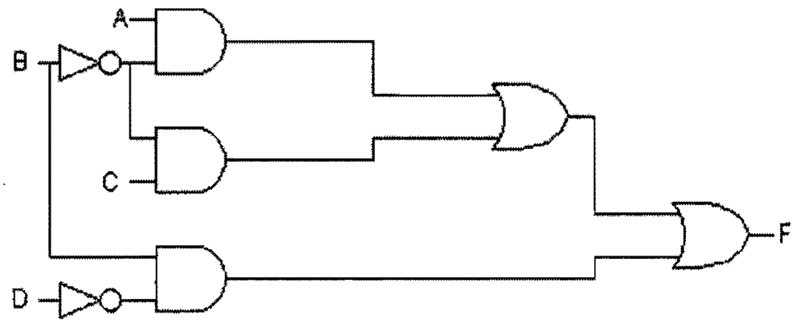
a) Digital circuits can be classified as either combination circuits or sequential circuits. Explain the differences between these circuits? Use diagrams in your explanation. [4]

b) A device accepts natural numbers in the range 0000 to 1111 that represent 0 to 15. The output F of the circuit is true if the input to the circuit represents a prime number and is false otherwise. [12]

  i) Draw the truth table for this function.
  ii) Hence, determine the canonical Sum of Products (SOP) and canonical Product of Sums (POS) expressions for the output F.
  iii) Write the short hand notation of the SOP and POS expressions.
  iv) Design a circuit using AND, OR and NOT gates to carry out this function.

c) Convert the following into SOP form and minimize using the Karnaugh map method.
$F = (AB+C)(B+\bar{C}D)$ [6]

d) Write down and simplify the logic function represented by the circuit diagram below: [3]



QUESTION 5

a) i) Briefly explain the difference between the Karnaugh map method and the Quine-McCluskey method. [3]

ii) Minimize the function $F(A,B,C,D) = \sum(0,1,2,3,6,7,8,9,14,15)$ using the Quine-McCluskey method. [10]

b) Flip flops can be implemented using R-S, D-type or J-K. Explain the different behaviors of these flip-flops. What additional logic is required to convert a J-K flip-flop into a D-type flip flop? [8]

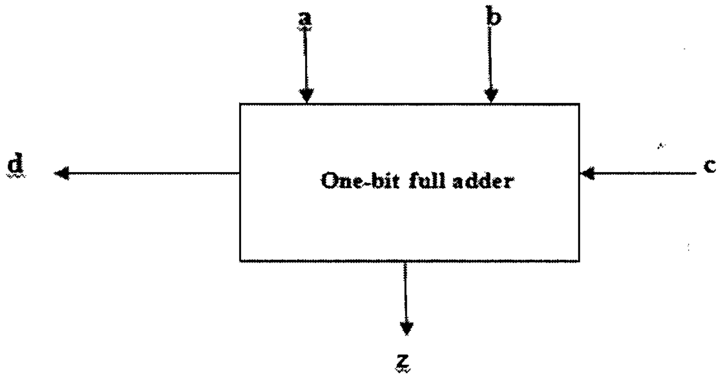
c) Simplify the following Boolean expressions using Boolean theorems. [4]

$$\overline{(A+B)\overline{CD} + \overline{F}}$$

QUESTION 6

(i) Using binary arithmetic evaluate the binary arithmetic expression: 101110+ 1101111 [4]

(ii) The binary ripple-carry addition algorithm used in (i), can be implemented using a one-bit full adder that takes two input bits **a** and **b**, and a carry-in bits **c** and compute a sum bit **z** and a carry-out bit **d.** The diagram below illustrates the input and outputs of a one-bit adder.



(a) Draw a truth table for the one-bit full-adder. [6]

(b) Based on the truth table obtained in (a) write the logical expressions for the sum bit **z** and the carry-out bit **d**. [4]

(c) Simplify the logical expressions obtained in (b), and draw one circuit diagram for the one-bit full adder. [6]

(iii) Using a diagram similar to one shown above for the one-bit adder, draw a diagram that illustrates how a 3-bit adder could be implemented using a combination of one-bit adders. [5]

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