University of Swaziland

Department of Computer Science

CS211 / CSC211 — Theory of Computation

Final Examination

December 2016

Instructions

1. Read all the questions in **Section A** and **Section B** before you start answering any question.
2. Answer **all** questions in Section A. Answer **any two** questions of Section B. Maximum mark is 100.
3. Use correct notation and show all your work on the answer script.

Section A

Question 1 [6 + 6 + 12]

The following languages are given on symbol set $\{a, b\}$. Assume that $u, v, w \in \{a, b\}^*$.

1. $L_1 = \{uwv, |u| = 2 \text{ and } |v| = 2\}$
2. $L_2 = \{awa\} \cup \{bwb\}$
3. $L_3 = \{w, (|w| \bmod 3 = 0)\}$

The following set of words is given -

{λ, a, b, ab, aab, abaa, aaabb, bbbbbab, aaabbbb, aaaaabb, aabbbbabb, ababab}

a From the above set write all words belonging to L_1, L_2 and L_3, respectively.

b Write the regular expressions representing L_1, L_2 and L_3 respectively.

c Design three deterministic finite acceptors (dfa's) accepting L_1, L_2 and L_3, respectively.

Question 2 [6 + 8 + 12]

You are given the following grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ and P is given by;

$S \rightarrow abA$,
$A \rightarrow baB$,
$B \rightarrow aA|bb$

a Does the grammar accept or reject the following words?

i *abbabb*

ii *ababbbb*

iii *abbaababb*

b Construct an NFA that accepts the language generated by the grammar above.

c Convert the NFA into a DFA.

Section B

Question 3 [25]

a **[6 + 6 + 1]**Given a context free grammar, $G = (\{S\}, \{a, b\}, S, P)$ where the set of productions P is given as

$$\{S \rightarrow aS|aSbS|\lambda\}$$

Write leftmost derivations for $w_1 = aaab$ and $w_2 = abab$. Taking examples of both w_1 and w_2, show that G is ambiguous by drawing two distinct parse trees for w_1 and w_2. What is the complexity of G.

b **[4 + 4 + 4]** Given the context free grammar, as ordered below, remove the following

$S \rightarrow aS|A|C|D$
$A \rightarrow aA|\lambda$
$B \rightarrow aa|\lambda$
$C \rightarrow aCb$
$D \rightarrow bD|b$

i λ productions.

ii unit productions.

iii useless productions.

Question 4 [25]

a **[10 + 5]** Design a deterministic pushdown automaton (**dpda**) to recognize the language—

$$L = \{w \in \{a, b\}^*,\ n_a(w)\ ?\ n_b(w), w \text{ always starts with an } a\}$$

Describe the functional steps of your **dpda**. Write instantaneous descriptions for $w = aabba$

b **[6 + 4]** Design a non deterministic pushdown automaton (**npda**) to recognize the language generated by the grammar in Griebach Normal Form—

$$G = (\{S, A, B\}, \{a, b\}, S, P)$$

where the set of productions P is —

{
$S \rightarrow aABB|aAA$
$A \rightarrow aBB|a$
$S \rightarrow bBB|aBB|a$
}

Write instantaneous descriptions of your **npda** for $w = aaabaa$.

Question 5 [15 + 5 + 5]

Write the functional steps of the design of a Turing Machine to compute:

$$F(x) = x \text{ div } 3$$

Assume x to be a non zero positive integer in unary representation. Also write the design and instantaneous descriptions using the values of x as 1111 and 1111111 (in unary representation) for your Turing Machine.