UNIVERSITY OF SWAZILAND

DEPARTMENT OF COMPUTER SCIENCE

CS211 / CSC211 — THEORY OF COMPUTATION

RE-SIT EXAMINATION

JULY 2018

Instructions

1. The time allowed is **THREE (3) HOURS**.
2. Read all the questions in **Section A** and **Section B** before you start answering any question.
3. Answer **all** questions in Section A. Answer **any two** questions of Section B. Maximum mark is 100.
4. Use correct notation and show all your work on the answer script.

DO NOT OPEN THIS PAPER UNTIL YOU ARE INSTRUCTED TO DO SO BY THE INVIGILATOR

Section A

Answer **all two** questions in this section.

Question 1 [25]

The following languages are given on symbol set $\{0, 1\}$. Assume that $u, v, w \in \{0, 1\}^*$.

i $L_1 = \{uwv : |u| = 2 \text{ and } |v| = 1\}$

ii $L_2 = \{0w0\} \cup \{1w1\}$

iii $L_3 = \{w : |w| \bmod 4 = 0\}$

The following set of words is given -
$\{\lambda, 0, 1, 01, 001, 0100, 00011, 1111101, 0001111, 00000011, 001111011, 010101\}$

a **[6]** From the above set, write all words belonging to L_1, all words belonging to L_2 and all words belonging to L_3.

b **[6]** Write regular expressions representing L_1, L_2 and L_3.

c **[12]** Design three deterministic finite acceptors accepting L_1, L_2 and L_3 respectively.

Question 2 [25]

The following non-deterministic finite accaptor is given:

$$M = (\{q_0, q_1, q_2\}, \{0, 1\}, q_0, \delta, \{q_1, q_2\})$$

where the transitions are gien as:
$\delta(q_0, 0) = \{q_0, q_1\};$
$\delta(q_1, 1) = \{q_0, q_2\};$
$\delta(q_2, 1) = \{q_1, q_2\};$

a **[6]** Draw the transition digraph of the above M.

b **[6]** Trace computations of all the words of L, where $L = \{111, 000, \text{ and } 010\}$

c **[14]** Find the equivalent **DFA** of M and write the state transition tale of your **DFA**.

Section B

Answer **any two** questions in this section.

Question 3 [25]

a A context free grammar, $G = (\{S\}, \{a, b\}, S, \{S \to aS|aSbS|\lambda\})$.

Using G, write leftmost derivations for $w_1 = aaaab$ and $w_2 = abab$. Taking examples of both w_1 and w_2, show that G is ambiguous by drawing two distinct parse trees for each w_1 and w_2. What is the complexity of G?

b **[9 + 6]** Assuming n, m and $k \geq 0$. Find Context Free Grammars G_1 and G_2 that generate the following languages.

i $L(G_1) = \{a^{2n}b^ic^{2m} : i = m + n\}$

ii $L(G_1) = \{a^nb^mc^{2k} : m = k\}$

Write leftmost derivations for $w_1 = aab, w_2 = bcc, w_3 = aabbcc$ using G_1 and $w_4 = bbcc, w_5 = aabbcc, w_6 = aabc$ using G_2.

Include production number at each step of your derivation.

Question 4 [25]

a **[10 + 5]** Design a deterministic pushdown automaton **(dpda)** to recognize the language—

$$L = \{a^nb^m : n > 0\}, m > n$$

Describe the functional steps of your **dpda**. Write instantaneous descriptions for $w_1 = aabbbb$ and $w_2 = aaabbbb$.

b **[6 + 4]** Design a non deterministic pushdown automaton **(npda)** to recognize the language generated by the grammar —

$$G = (\{S, A, B, C\}, \{a, b, c\}, S, P)$$

where the set of productions P is — {
$S \to aA$
$A \to bB|aABC|a$
$B \to bB|b$
$C \to cC|c$
}
Write instantaneous descriptions of your **npda** for $w = aaabc$.

Question 5 [15 + 5 + 5]

Write the design of a Turing Machine (TM) to compute:

$$F(x) = x \text{ div } 2.$$

Assume x to be a non zero positive integer in unary representation. Clearly write the functional steps of your TM computations. Also write the instantaneous descriptions using the values of x as 11 and 1111 (in unary representation) for your Turing Machine.