UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF COMPUTER SCIENCE

MAIN EXAMINATION

DECEMBER 2018

TITLE OF PAPER: INTRODUCTION TO LOGIC

COURSE CODE: CSC201

TIME ALLOWED: 3 HOURS **TOTAL MARKS: 100**

INSTRUCTIONS TO CANDIDATES:

1. All questions carry equal marks.
2. **Question 1 is compulsory.**
3. Answer any 3 Questions from Question 2 to Question 5.
4. Marks for each question are indicated in square brackets.
5. Show all your workings where necessary.

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

Question 1

(a) State if the following statements are true or false. [4]

(i) Every argument with a false conclusion is unsound.

(ii) Every argument that is valid and has a true conclusion is sound.

(iii) Every valid argument has at least one true premise.

(iv) Every argument with all true premises and a false conclusion is invalid.

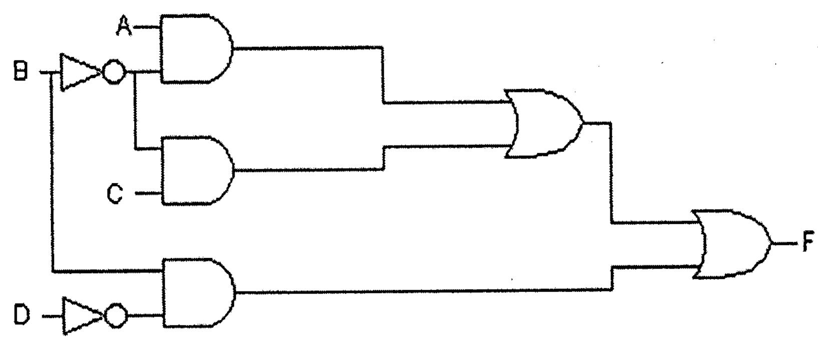
(b) Represent the following statements in propositional logic syntax. [4]

(i) I'll get a bicycle if and only if the city builds a bike path and I don't buy either a new or a used car.

(ii) I won't buy both a bicycle and a used car unless I either get a raise or inherit a lot of money.

(c) What do we mean by sufficiency set in digital logic? Give an example. [2]

(d) What is the function represented by this digital circuit? Simplify it if possible. [4]



(e) Consider the set of integer numbers, and let

N(x): x is an integer

E(x): x is even

O(x): x is odd

P(x): x is prime

Translate the following into predicate logical notation

(i) There exists an even integer number [1]

(ii) The is at least one even prime number [2]

(iii)If integer x is even, then 2x is even [2]

(f) Prove the equivalence elimination law using a truth table. [6]

Question 2

(a) Check if the following statements are consistent with each other. [5]

- $A \wedge B$
- $C \rightarrow \neg B$
- C

(b) Using laws of logical equivalence, verify the following statements.

(i) $((A \wedge \neg B) \rightarrow (C \vee A)) \cong B \vee C$ [4]

(ii) $(R \rightarrow \neg P) \leftrightarrow (P \vee R)$ *is a tautology* [4]

(iii) $((P \wedge \neg Q) \vee (P \wedge R)) \wedge (Q \rightarrow P)$ *entails* P [4]

(c) Find the DNF and CNF of $(A \wedge \neg B) \rightarrow (C \vee A))$ [6]

(d) Give an example of a valid but unsound argument. [2]

Question 3

(a) Is the following argument valid? Show your working. [4]

$$A \vee B$$

$$A \rightarrow C$$

$$B \rightarrow C$$

$$\therefore C$$

(b) Use resolution rule to prove that the following premises

- $\neg s \wedge c$
- $w \rightarrow s$
- $\neg w \rightarrow t$
- $t \rightarrow h$

lead to the conclusion

- h. [8]

(c) Implement the circuit of the following function using NAND gates only. [8]

$$f(a,b,c) = a + a \cdot \bar{b} + \overline{bc}$$

(d) Draw the truth table of the digital function above. [5]

Question 4

(a) List two differences between sequential and combinational circuits? [2]

(b) Minimize the function $F(A,B,C,D)=\sum(0,1,2,3,6,7,8,9,14,15)$ using the Quine-McCluskey method. [10]

(c) Design a reduced circuit that inputs a 4 bit number. The circuit outputs 1 if the number is greater than 8 otherwise it outputs a zero. [10]

(d) What are the advantages of having a clock on a combinational circuit? [3]

Question 5

(a) Prove the following logical equivalence. [5]

$$\neg\forall x\left(\exists y(\neg P(x,y)) \rightarrow \forall x(Q(x))\right) \equiv \exists x\left(\exists y(\neg P(x,y)) \wedge \exists x(\neg Q(x))\right)$$

(b) Remove the existential quantifier. [4]

$$\exists x(\text{Loves}(x, \text{Ben})) \wedge \exists y(\neg\text{Loves}(y, \text{Ben}))$$

(c) Draw the reduced circuit that implements the following function. [10]

$$f(a,b,c,d) = \bar{a}b\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}bc\bar{d} + abc\bar{d} + a\bar{b}\bar{c}d + a\bar{b}cd$$

(e) What are the advantages of the quine McCluskey method over the k-map method? [2]

(f) Explain with examples the difference between inductive and deductive reasoning. [4]